

Filosofie e culture dal medioevo al rinascimento
Marco Forlivesi

The Medieval Geometry of Qualities and Motions: Philosophy and Mathematics in the Late Middle Ages
Daniel A. Di Liscia

Dear participants,

This is the list of the texts we are going to work with in our course. The texts themselves are available in separate pdf-files. Let me now add some remarks you could find useful:

1. The materials were organised according to the three working days: Tuesday, Wednesday and Thursday. It would be very useful, if you found the time to take a look at them before the lecturing day.
2. In the attached pdf-files you will find the reference TEXT 1, TEXT 2, etc. at the top of the pages, matching the numbering of the list (same names for the files).
3. None of these texts are a complete text; they are rather representative passages of different works connected thematically.
4. To make possible a better understanding of the technical terminology and of the argumentation strategies I decided to include with one exception the originals – mostly Latin – of the proposed translations.
5. I translated text 5 for this course. It is a passage of a Latin text I have edited in my contribution “Walter Burley, Paulus Venetus und die Tradition *De instanti* (mit dem *Tractatus de instanti* des Paulus Venetus nach Hs. Florenz, Biblioteca Nazionale Centrale, II. IV. 553, ff. 68^va-69^va)”, in: A. Speer - D. Wirmer (eds.), *Das Sein der Dauer*, [Miscellanea Mediaevalia 34], Berlin-New York, 2008, pp. 123-150.
6. The short texts 12 were edited in my contribution “The *Latitudines breves* and Late Medieval University Teaching”, *SCIAMVS* 17 (2016), pp. 55-120 (Lat. and Engl. in the same file 12), I have cut them out from there.
7. Texts 1-2, 6-8-7-12 (in this order) and 9-10 are scanned respectively in one file together.

Date	Author	Text
Day 1 Tuesday (Oct. 24)		A) Historical Background
	1) Aristotle	<i>Physics</i> , II. 2, 193b22 – 194b15
	2) T. Aquinas	<i>Commentary on Aristotle's Physics</i> , lect. 3.
	3) R. Grosseteste	<i>Concerning lines, angles and figures</i>
		B) Calculatores
	4) T. Bradwardine	<i>Treatise on the Proportions</i> ...Foreword
	5) W. Burley/P. Venetus	<i>Question / Treatise on the instant</i>
	6) W. Heytesbury	<i>On solving sophismata</i>
	7) (Pseudo?)-W. Heytesbury	<i>Probationes conclusionum</i>
	8) Johannes Hollandrinus	<i>On motion</i>
Day 2 Wednesday (Oct. 25)		Configurations and latitudes
	9) Nicole Oresme	<i>Treatise on the configuration of qualities and motions</i>
	10) Jacobus de S. Martino(?)	<i>Treatise on the latitude of the forms</i>
Day 3 Thursday (Oct. 26)		The Reception in the 14th, 15th and 17th centuries
	11) Anonymous	<i>Latitudines breves</i> 1, 2, 3, forewords
	12) Biagio Pelacani da Parma	<i>Questions of the "Treatise on the latitude of forms"</i>
	13) Galileo Galilei	<i>Discorsi</i> , Giorn. III, th. 1

Munich, 15 October 2017
Daniel A. Di Liscia



Commentaria in octo libros Physicorum
COMMENTARY ON ARISTOTLE'S PHYSICS

by
Thomas Aquinas
Books I-II translated by
Richard J. Blackwell, Richard J. Spath & W. Edmund Thirlkel
Yale U.P., 1963

Books III-VIII translated by
Pierre H. Conway, O.P.
College of St. Mary of the Springs, Columbus, Ohio
1958-1962

<http://www.dhspriory.org/thomas/Physics.htm>

Lectio 3 (193 b 22-194 a 11)
HOW PHYSICS AND MATHEMATICS DIFFER IN THEIR CONSIDERATION OF THE SAME THING

Chapter 2 cont.

Ἐπεὶ δὲ διώρισται ποσαχῶς ἡ φύσις, μετὰ τοῦτο θεωρητέον τίνι διαφέρει ὁ μαθηματικὸς τοῦ φυσικοῦ

We have distinguished, then, the different ways in which the term 'nature' is used. The next point to consider is how the mathematician differs from the physicist.

(καὶ γὰρ ἐπίπεδα καὶ στερεὰ ἔχει τὰ φυσικὰ σώματα καὶ μήκη καὶ στιγμάς, περὶ ὧν σκοπεῖ ὁ μαθηματικός).

Obviously physical bodies contain surfaces and volumes, lines and points, and these are the subject-matter of mathematics.

ἔτι εἰ ἡ ἀστρολογία ἑτέρα ἢ μέρος τῆς φυσικῆς· εἰ γὰρ τοῦ φυσικοῦ τὸ τί ἐστὶν ἥλιος ἢ σελήνη εἰδέναι, τῶν δὲ συμβεβηκότων καθ' αὐτὰ μηδέν, ἄτοπον, ἄλλως τε καὶ ὅτι φαίνονται λέγοντες οἱ περὶ φύσεως καὶ περὶ σχήματος σελήνης καὶ ἡλίου, καὶ δὴ καὶ πότερον σφαιροειδὴς ἢ γῆ καὶ ὁ κόσμος ἢ οὐ.

Further, is astronomy different from physics or a department of it? It seems absurd that the physicist should be supposed to know the nature of sun or moon, but not to know any of their essential attributes, particularly as the writers on physics obviously do discuss their shape also and whether the earth and the world are spherical or not.

περὶ τούτων μὲν οὖν πραγματεύεται καὶ ὁ μαθηματικός, ἀλλ' οὐχ ἡ φυσικοῦ σώματος πέρας ἕκαστον· οὐδὲ τὰ συμβεβηκότα θεωρεῖ ἢ τοιούτοις οὔσι συμβέβηκεν.

Now the mathematician, though he too treats of these things, nevertheless does not treat of them as the limits of a physical body; nor does he consider the attributes indicated as the attributes of such bodies.

διὸ καὶ χωρίζει· χωριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι, καὶ οὐδὲν διαφέρει, οὐδὲ γίγνεται ψεῦδος χωριζόντων.

That is why he separates them; for in thought they are separable from motion, and it makes no difference, nor does any falsity result, if they are separated.

λανθάνουσι δὲ τοῦτο ποιοῦντες καὶ οἱ τὰς ἰδέας λέγοντες· τὰ γὰρ φυσικὰ (194a.) χωρίζουσιν ἥττον ὄντα χωριστὰ τῶν μαθηματικῶν.

The holders of the theory of Forms do the same, though they are not aware of it; for they separate the objects of physics, which are less separable than those of mathematics.

γίγνωιτο δ' ἂν τοῦτο δῆλον, εἴ τις ἐκατέρων πειρώτω λέγειν τοὺς ὄρους, καὶ αὐτῶν καὶ τῶν συμβεβηκότων. τὸ μὲν γὰρ περιττὸν ἔσται καὶ

This becomes plain if one tries to state in each of the two cases the definitions of the things and of their attributes. 'Odd' and 'even', 'straight' and

τὸ ἄρτιον καὶ τὸ εὐθύ καὶ τὸ καμπύλον, ἔτι δὲ ἀριθμὸς καὶ γραμμὴ καὶ σχῆμα, ἄνευ κινήσεως, σὰρξ δὲ καὶ ὁστοῦν καὶ ἄνθρωπος οὐκέτι, ἀλλὰ ταῦτα ὥσπερ ῥίς σιμὴ ἀλλ' οὐχ ὡς τὸ καμπύλον λέγεται.

δηλοῖ δὲ καὶ τὰ φυσικώτερα τῶν μαθημάτων, οἷον ὀπτική καὶ ἁρμονική καὶ ἀστρολογία· ἀνάπαλιν γὰρ τρόπον τιν' ἔχουσιν τῇ γεωμετρίας· ἡ μὲν γὰρ γεωμετρία περὶ γραμμῆς φυσικῆς σκοπεῖ, ἀλλ' οὐχ ἡ φυσική, ἡ δ' ὀπτική μαθηματικὴν μὲν γραμμὴν, ἀλλ' οὐχ ἡ μαθηματικὴ ἀλλ' ἡ φυσική.

'curved', and likewise 'number', 'line', and 'figure', do not involve motion; not so 'flesh' and 'bone' and 'man'—these are defined like 'snub nose', not like 'curved'.

Similar evidence is supplied by the more physical of the branches of mathematics, such as optics, harmonics, and astronomy. These are in a way the converse of geometry. While geometry investigates physical lines but not qua physical, optics investigates mathematical lines, but qua physical, not qua mathematical.

Postquam philosophus ostendit quid sit natura et quot modis dicitur, hic consequenter intendit ostendere de quibus considerat scientia naturalis.

Et dividitur in partes duas: in prima ostendit quomodo differat naturalis a mathematico; in secunda ostendit ad quae se extendat consideratio scientiae naturalis, ibi: quoniam autem natura et cetera.

Circa primum tria facit: primo movet quaestionem; secundo ponit rationes ad quaestionem, ibi: etenim plana etc.; tertio solvit quaestionem, ibi: de his igitur negotiatur et cetera.

Dicit ergo primo quod postquam determinatum est quot modis natura dicitur, considerandum est in quo differat mathematicus a naturali philosopho.

Deinde cum dicit: etenim plana etc., ponit rationes ad quaestionem. Quarum prima talis est..

Quaecumque scientiae considerant eadem subiecta, vel sunt eadem, vel una est pars alterius; sed mathematicus philosophus considerat de punctis, lineis et superficiebus et corporibus, et similiter naturalis (quod probat ex hoc quod corpora naturalia habent plana, idest superficies, et firma, idest soliditates, et longitudines et puncta; oportet autem quod naturalis consideret de omnibus quae insunt

157. After the Philosopher has explained what nature is and how many ways the name is used, he here intends to show what it is that natural science considers.

This section is divided into two parts. First he shows how natural science differs from mathematics. Secondly, where he says, 'Since nature has...' (194 a 12; L4 #166), he designates that to which the consideration of natural science extends.

Concerning the first part he makes three points. First he states the question. Secondly, where he says, 'Obviously physical bodies ...' (193 b 23 #158), he gives his reasons for [raising] the question. Thirdly, he answers the question where he says, 'Now the mathematician...' (193 b 31 #159).

He says, therefore, first that after the uses of the name 'nature' have been determined, it is necessary to consider how mathematics differs from natural philosophy.

158. Next where he says, 'Obviously physical bodies ...' (193 b 23), he gives his reasons for [raising] the question. The first of these is as follows.

Whenever sciences consider the same subjects, they are either the same science, or one is a part of the other. But the mathematical philosopher considers points and lines and surfaces and bodies, and so does the natural philosopher. (For he proves from the fact that natural bodies have planes, i.e., surfaces, and volumes, i.e., solidity, and lengths and points. Moreover the natural

TEXTS N° 1 and N° 2

corporibus naturalibus); ergo videtur quod scientia naturalis et mathematica vel sint eadem, vel una sit pars alterius

Secundam rationem ponit ibi: amplius, astrologia et cetera. Et circa hanc rationem primo movet quaestionem, utrum astrologia sit omnino altera a naturali philosophia, vel sit pars eius. Manifestum est enim quod est pars mathematicae: unde si est etiam pars naturalis philosophiae, sequitur quod mathematica et physica conveniunt ad minus in hac parte.

Quod autem astrologia sit pars physicae, probat dupliciter. Primo quidem per rationem talem. Ad quemcumque pertinet cognoscere substantias et naturas aliquarum rerum, ad eum etiam pertinet considerare accidentia illarum; sed ad naturalem pertinet considerare naturam et substantiam solis et lunae, cum sint quaedam corpora naturalia; ergo ad eum pertinet etiam considerare per se accidentia ipsorum.

Hoc etiam probat ex consuetudine philosophorum: nam philosophi naturales inveniuntur determinasse de figura solis et lunae et terrae et totius mundi, circa quod insudat etiam astrologorum intentio. Sic igitur astrologia et scientia naturalis conveniunt non solum in eisdem subiectis, sed etiam in consideratione eorundem accidentium, et in demonstratione earundem conclusionum. Unde videtur quod astrologia sit pars physicae; et per consequens physica non totaliter differat a mathematica.

Deinde cum dicit: de his quidem igitur etc., solvit praemissam quaestionem. Et circa hoc duo facit: primo ponit solutionem; secundo confirmat eam, ibi: fiet autem utique et cetera.

Circa primum tria facit: primo solvit quaestionem; secundo concludit quoddam corollarium ex praedictis, ibi: unde et abstrahit etc.; tertio excludit errorem, ibi: latet autem hoc et cetera.

Dicit ergo primo quod mathematicus et naturalis determinant de eisdem, scilicet punctis, lineis et

philosopher must consider all things that are in natural bodies.) Therefore it seems that natural science and mathematics are either the same or that one is a part of the other.

He gives the second reason where he says, 'Further, is astronomy...' (193 b 25). In connection with this reason he raises the question whether astronomy is altogether other than natural philosophy or a part of it. For it is clear that astronomy is a part of mathematics. Whence, if it is also a part of natural philosophy, it follows that mathematics and physics agree at least in this part.

That astronomy is a part of physics he proves in two ways. First by the following argument. To whomever it belongs to know the substances and natures of certain things, also belongs the consideration of their accidents. But it belongs to the natural philosopher to consider the nature and substance of the sun and the moon, since they are certain natural bodies. Therefore it belongs to the natural philosopher to consider their per se accidents.

He proves this also from the custom of the philosophers. For natural philosophers are found to have treated the shape of the sun and of the moon and of the earth and of the whole world. And these are topics which claim the attention of the astronomers. Therefore astronomy and natural science agree not only in [having] the same subjects but also in the consideration of the same accidents, and in demonstrating the same conclusions. Whence it seems that astronomy is a part of physics, and as a result physics does not differ totally from mathematics.

159. Next where he says, 'Now the mathematician ...' (193 b 3 1), he answers the question raised above. Concerning this he makes two points. First he gives his solution, and secondly he confirms it, where he says, 'This becomes plain...' (194 a 1 #163).

Concerning the first part he makes three points. First he answers the question. Secondly, where he says, 'That is why he separates ...' (193 b 33 #161), he concludes to a sort of corollary from the above. Thirdly, where he says, 'The holders of ...' (193 b 35 #162), he excludes an error.

160. He says, therefore, first that the mathematician and the natural philosopher treat

superficiebus et huiusmodi, sed non eodem modo. Non enim mathematicus determinat de eis inquantum unumquodque eorum est terminus corporis naturalis; neque considerat ea quae accidunt eis inquantum sunt termini corporis naturalis; per quem modum de eis considerat scientia naturalis. Non est autem inconueniens quod idem cadat sub consideratione diversarum scientiarum secundum diversas considerationes.

Deinde cum dicit: unde et abstrahit etc., concludit quoddam corollarium ex praedictis. Quia enim mathematicus considerat lineas et puncta et superficies et huiusmodi et accidentia eorum non inquantum sunt termini corporis naturalis, ideo dicitur abstrahere a materia sensibili et naturali. Et causa quare potest abstrahere, est ista: quia secundum intellectum sunt abstracta a motu.

Ad cuius causae evidentiam considerandum est quod multa sunt coniuncta secundum rem, quorum unum non est de intellectu alterius: sicut album et musicum coniunguntur in aliquo subiecto, et tamen unum non est de intellectu alterius, et ideo potest unum separatim intelligi sine alio. Et hoc est unum intellectum esse abstractum ab alio. Manifestum est autem quod posteriora non sunt de intellectu priorum, sed e converso: unde priora possunt intelligi sine posterioribus, et non e converso. Sicut patet quod animal est prius homine, et homo est prius hoc homine (nam homo se habet ex additione ad animal, et hic homo ex additione ad hominem); et propter hoc homo non est de intellectu animalis, nec Socrates de intellectu hominis: unde animal potest intelligi absque homine, et homo absque Socrate et aliis individuus. Et hoc est abstrahere universale a particulari.

Similiter autem inter accidentia omnia quae adveniunt substantiae, primo advenit ei quantitas, et deinde qualitates sensibiles et actiones et passionem et motus consequentes sensibiles qualitates. Sic igitur quantitas non claudit in sui intellectu qualitates

the same things, i.e., points, and lines, and surfaces, and things of this sort, but not in the same way. For the mathematician does not treat these things insofar as each of them is a boundary of a natural body, nor does he consider those things which belong to them insofar as they are the boundaries of a natural body. But this is the way in which natural science treats them. And, it is not inconsistent that the same thing should fall under the consideration of different sciences according to different points of view.

161. Next where he says, 'That is why he separates ...' (193 b 33), he concludes to a sort of corollary from what he has just said. Because the mathematician does not consider lines, and points, and surfaces, and things of this sort, and their accidents, insofar as they are the boundaries of a natural body, he is said to abstract from sensible and natural matter. And the reason why he is able to abstract is this: according to the intellect these things are abstracted from motion.

As evidence for this reason we must note that many things are joined in the thing, but the understanding of one of them is not derived from the understanding of another. Thus white and musical are joined in the same subject, nevertheless the understanding of one of these is not derived from an understanding of the other. And so one can be separately understood without the other. And this one is understood as abstracted from the other. It is clear, however, that the posterior is not derived from the understanding of the prior, but conversely. Hence the prior can be understood without the posterior, but not conversely. Thus it is clear that animal is prior to man, and man is prior to this man (for man is had by addition to animal, and this man by addition to man). And because of this our understanding of man is not derived from our understanding of animal, nor our understanding of Socrates from our understanding of man. Hence animal can be understood without man, and man without Socrates and other individuals. And this is to abstract the universal from the particular.

In like manner, among all the accidents which come to substance, quantity comes first, and then the sensible qualities, and actions and passions, and the motions consequent upon sensible qualities. Therefore quantity does not embrace in

sensibiles vel passiones vel motus: claudit tamen in sui intellectu substantiam. Potest igitur intelligi quantitas sine materia subiecta motui et qualitatibus sensibilibus, non tamen absque substantia. Et ideo huiusmodi quantitates et quae eis accidunt, sunt secundum intellectum abstracta a motu et a materia sensibili, non autem a materia intelligibili, ut dicitur in VII Metaphys.

Quia igitur sic sunt abstracta a motu secundum intellectum, quod non claudunt in suo intellectu materiam sensibilem subiectam motui; ideo mathematicus potest ea abstrahere a materia sensibili. Et nihil differt quantum ad veritatem considerationis, utrum sic vel sic considerentur. Quamvis enim non sint abstracta secundum esse, non tamen mathematici abstrahentes ea secundum intellectum, mentiuntur: quia non asserunt ea esse extra materiam sensibilem (hoc enim esset mendacium), sed considerant de eis absque consideratione materiae sensibilis, quod absque mendacio fieri potest: sicut aliquis potest considerare albedinem absque musica, et vere, licet conveniant in eodem subiecto: non tamen esset vera consideratio, si assereret album non esse musicum.

Deinde cum dicit: latet autem hoc facientes etc., excludit ex praedictis errorem Platonis.

Quia enim latebat eum quomodo intellectus vere posset abstrahere ea quae non sunt abstracta secundum esse, posuit omnia quae sunt abstracta secundum intellectum, esse abstracta secundum rem. Unde non solum posuit mathematica abstracta, propter hoc quod mathematicus abstrahit a materia sensibili; sed etiam posuit ipsas res naturales abstractas, propter hoc quod naturalis scientia est de universalibus et non de singularibus. Unde posuit hominem esse separatum, et equum et lapidem et alia huiusmodi; quae quidem separata dicebat esse ideas: cum tamen naturalia sint minus abstracta quam mathematica. Mathematica enim sunt omnino abstracta a materia sensibili secundum intellectum, quia materia sensibilis non includitur in intellectu mathematicorum, neque in universali neque in particulari: sed in intellectu specierum naturalium includitur quidem materia sensibilis, sed non materia

its intelligibility the sensible qualities or the passions or the motions. Yet it does include substance in its intelligibility. Therefore quantity can be understood without matter, which is subject to motion, and without sensible qualities, but not without substance. And thus quantities and those things which belong to them are understood as abstracted from motion and sensible matter, but not from intelligible matter, as is said in Metaphysics, VII:10.

Since, therefore, the objects of mathematics are abstracted from motion according to the intellect, and since they do not include in their intelligibility sensible matter, which is a subject of motion, the mathematician can abstract them from sensible matter. And it makes no difference as far as the truth is concerned whether they are considered one way or the other. For although the objects of mathematics are not separated according to existence, the mathematicians, in abstracting them according to their understanding, do not lie, because they do not assert that these things exist apart from sensible matter (for this would be a lie). But they consider them without any consideration of sensible matter, which can be done without lying. Thus one can truly consider the white without the musical, even though they exist together in the same subject. But it would not be a true consideration if one were to assert that the white is not musical.

162. Next where he says, "The holders of the theory..." (193 b 35), he excludes from what he has said an error of Plato.

Since Plato was puzzled as to how the intellect could truly separate those things which were not separated in their existence, he held that all things which are separated in the understanding are separated in the thing. Hence he not only held that mathematical entities are separated, because of the fact that the mathematician abstracts from sensible matter, but he even held that natural things themselves are separated, because of the fact that natural science is of universals and not of singulars. Hence he held that man is separated, and horse, and stone, and other such things. And he said these separated things are ideas, although natural things are less abstract than mathematical entities. For mathematical entities are altogether separated from sensible matter in the understanding, because sensible matter is not included in the

TEXTS N° 1 and N° 2

individualis; in intellectu enim hominis includitur caro et os, sed non haec caro et hoc os.

Deinde cum dicit: fiet autem utique manifestum etc., manifestat positam solutionem dupliciter: primo quidem per differentiam definitionum quas assignat mathematicus et naturalis; secundo per scientias medias, ibi: demonstrant autem et quae magis et cetera.

Dicit ergo primo quod hoc quod dictum est de diverso modo considerationis mathematici et physici, fiet manifestum si quis tentaverit dicere definitiones naturalium et mathematicorum, et accidentium eorum: quia mathematica, ut par et impar, et rectum et curvum, et numerus et linea et figura, definiuntur sine motu et materia; non autem caro et os et homo: sed horum definitio est sicut definitio simi, in cuius definitione ponitur subiectum sensibile, scilicet nasus; non autem sicut definitio curvi, in cuius definitione non ponitur aliquod subiectum sensibile.

Et sic ex ipsis definitionibus naturalium et mathematicorum apparet quod supra dictum est de differentia mathematici et naturalis.

Deinde cum dicit: demonstrant autem etc., probat idem per scientias quae sunt mediae inter mathematicam et naturalem.

Dicuntur autem scientiae mediae, quae accipiunt principia abstracta a scientiis pure mathematicis, et applicant ad materiam sensibilem; sicut perspectiva applicat ad lineam visualemente ea quae demonstrantur a geometria circa lineam abstractam; et harmonica, id est musica, applicat ad sonos ea quae arithmeticus considerat circa proportionem numerorum; et astrologia considerationem geometriae et arithmeticae applicat ad caelum et ad partes eius.

understanding of the mathematical, neither in the universal nor in the particular. But sensible matter is included in the understanding of natural things, whereas individual matter is not. For in the understanding of man flesh and bone is included, but not this flesh and this bone.

163. Next where he says, 'This becomes plain ...' (194 a 1), he clarifies the solution he has given in two ways, first by means of the difference in the definitions which the mathematician and the natural philosopher assign, and secondly by means of the intermediate sciences, where he says, 'Similar evidence ...' (194 a 7 #164).

He says, therefore, first that what has been said of the different modes of consideration of the mathematician and the natural philosopher will become evident if one attempts to give definitions of the mathematical, and of natural things and of their accidents. For the mathematical, such as equal and unequal, straight and curved, and number, and line, and figure, are defined without motion and matter, but this is not so with flesh and bone and man. Rather the definition of these latter is like the definition of the snub in which definition a sensible subject is placed, i.e., nose. But this is not the case with the definition of the curved in which definition a sensible subject is not placed.

And thus from the very definitions of natural things and of the mathematical, what was said above [#160ff] about the difference between the mathematician and the natural philosopher is apparent.

164. Next where he says, 'Similar evidence...' (194 a 7), he proves the same thing by means of those sciences which are intermediates between mathematics and natural philosophy.

Those sciences are called intermediate sciences which take principles abstracted by the purely mathematical sciences and apply them to sensible matter. For example, perspective applies to the visual line those things which are demonstrated by geometry about the abstracted line; and harmony, that is music, applies to sound those things which arithmetic considers about the proportions of numbers; and astronomy applies the consideration of geometry and arithmetic to the heavens and its parts.

Huiusmodi autem scientiae, licet sint mediae inter scientiam naturalem et mathematicam, tamen dicuntur hic a philosopho esse magis naturales quam mathematicae, quia unumquodque denominatur et speciem habet a termino: unde, quia harum scientiarum consideratio terminatur ad materiam naturalem, licet per principia mathematica procedant, magis sunt naturales quam mathematicae.

Dicit ergo de huiusmodi scientiis, quod contrario modo se habent cum scientiis quae sunt pure mathematicae, sicut geometria vel arithmetica. Nam geometria considerat quidem de linea quae habet esse in materia sensibili, quae est linea naturalis: non tamen considerat de ea in quantum est in materia sensibili, secundum quod est naturalis, sed abstracte, ut dictum est. Sed perspectiva e converso accipit lineam abstractam secundum quod est in consideratione mathematici, et applicat eam ad materiam sensibilem; et sic determinat de ea non in quantum est mathematica, sed in quantum est physica.

Ex ipsa ergo differentia scientiarum mediarum ad scientias pure mathematicas, apparet quod supra dictum est. Nam si huiusmodi scientiae mediae abstracta applicant ad materiam sensibilem, manifestum est quod mathematicae e converso ea quae sunt in materia sensibili abstrahunt.

Et per hoc etiam patet responsio ad id quod supra obiiciebatur de astrologia. Unde astrologia est magis naturalis quam mathematica. Unde non est mirum si communicet in conclusionibus cum scientia naturali.

Quia tamen non est pure naturalis, per aliud medium eandem conclusionem demonstrat. Sicut quod terra sit sphaerica demonstratur a naturali per medium naturale, ut puta quia partes eius undique et aequaliter concurrunt ad medium: ab astrologo autem ex figura eclipsis lunaris, vel ex hoc quod non eadem sidera ex omni parte terrae aspiciuntur.

However, although sciences of this sort are intermediates between natural science and mathematics, they are here said by the Philosopher to be more natural than mathematical, because each thing is named and takes its species from its terminus. Hence, since the consideration of these sciences is terminated in natural matter, then even though they proceed by mathematical principles, they are more natural than mathematical sciences.

He says, therefore, that sciences of this sort are established in a way contrary to the sciences which are purely mathematical, such as geometry or arithmetic. For geometry considers the line which has existence in sensible matter, which is the natural line. But it does not consider it insofar as it is in sensible matter, insofar as it is natural, but abstractly, as was said [160ff]. But perspective conversely takes the abstract line which is in the consideration of mathematics, and applies it to sensible matter, and thus treats it not insofar as it is a mathematical, but insofar as it is a physical thing.

Therefore from this difference between intermediate sciences and the purely mathematical sciences, what was said above is clear. For if intermediate sciences of this sort apply the abstract to sensible matter, it is clear that mathematics conversely separates those things which are in sensible matter.

165. And from this it is clear what his answer is to the objection raised above [158] concerning astronomy. For astronomy is a natural science more than a mathematical science. Hence it is no wonder that astronomy agrees in its conclusions with natural science.

However, since it is not a purely natural science, it demonstrates the same conclusion through another method. Thus, the fact that the earth is spherical is demonstrated by natural science by a natural method, e.g., because its parts everywhere and equally come together at the middle. But this is demonstrated by astronomy from the figure of the lunar eclipse, or from the fact that the same stars are not seen from every part of the earth.

3. Robert Grosseteste

From E. Grant (ed.) *A Source Book in Medieval Science* (Cambridge, Massachusetts: Harvard Univ. Press, 1974), pp. 384-85.

1. Robert Grosseteste (ca. 1168–1253): Concerning Lines, Angles, and Figures⁵

The usefulness of considering lines, angles, and figures is very great, since it is impossible to understand natural philosophy without them. They are useful in relation to the universe as a whole and its individual parts. They are useful also in connection with related properties, such as rectilinear and circular motion. Indeed, they are useful in relation to activity and receptivity, whether of matter or sense; and if the latter, whether of the sense of vision, where activity and receptivity are apparent, or of the other senses, in the operation of which something must be added to those things that produce vision.

Since we have spoken elsewhere of those things that pertain to the whole universe and its individual parts and of those things that relate to rectilinear and circular motion,⁶ we must now consider universal action insofar as it partakes of the nature of sublunary things (*inferiorum*); this universal action is a subject receptive to diverse activities, insofar as it descends to operation in the matter of the world; and some things can be brought in as intermediaries, which are able to bring to perfection that which is advancing toward greater things (*maiora*).

Now, all causes of natural effects must be expressed by means of lines, angles, and figures, for otherwise it is impossible to grasp their explanation. This is evident as follows. A natural agent multiplies its power from itself to the recipient, whether it acts on sense or on matter. This power is sometimes called species, sometimes a likeness, and it is the same thing whatever it may be called; and the agent sends the same power into sense and into matter, or into its own contrary, as

heat sends the same thing into the sense of touch and into a cold body. For it does not act by deliberation and choice, and therefore it acts in a single

manner whatever it encounters, whether sense or something insensitive, whether something animate or inanimate. But the effects are diversified by the diversity of the recipient, for when this power is received by the senses, it produces an effect that is somehow spiritual and noble; on the other hand, when it is received by matter, it produces a material effect. Thus the sun produces different effects in different recipients by the same power, for it cakes mud and melts ice.

The power from a natural agent reaches the recipient either along a shorter line—and then the power is more active, since the recipient is less distant from the agent—or along a longer line—and then it is less active, since the recipient is more distant. In any case, the power comes from the surface of the agent with or without mediation. If without mediation, the power comes either by a straight line or by a bent line. But if by a straight line, the action is stronger and better, as Aristotle proposes in Book V of the *Physics*,⁷ since nature acts in the briefest possible manner; and a straight line is the shortest of all lines, as he says in the same place. Moreover, a straight line possesses equality because it has no angle; but the equal is superior to the unequal, as Boethius says in his *Arithmetic*.⁸ However, nature acts in the briefest possible manner; consequently it acts best along a straight line. Similarly, every united power acts more strongly. But there is greater oneness and unity in a straight line than in a bent line, as is asserted in Book V of the *Metaphysics*;⁹ therefore action in a straight line is stronger.

Transl. from *Die philosophischen Werke des Robert Grosseteste*, Bischofs von Lincoln, ed. L. Bauer, in: *Beiträge zur Geschichte der Philosophie des Mittelalters* IX (1912), pp. 59-65.

THOMAS BRADWARDINI TRACTATUS PROPORTIONUM SEU DE
PROPORTIONIBUS VELOCITATUM IN MOTIBUS

PROEMIUM

Omnem motum successivum alteri in velocitate proportionari
conferat; quapropter philosophia naturalis, quae de motu con-
siderat, proportionem motum et velocitatum in motibus ignorare
non debet. Et quia cognitio illius est necessaria et multum difficilis,
nec in aliqua parte philosophiae tradita est ad plenum, ideo de pro-
portione velocitatum in motibus fecimus istud opus. Et quia, tes-
tante Boethio, primo *Arithmeticae* suae: Quisquis scientias mathe-
maticales praefermiserit, constat eum omnem philosophiae per-
didisse doctrinam, ideo mathematica quibus ad propositum
indigemus praemisimus, ut sit doctrina facillior et promptior
inquirenti. Et propter maiorem promptitudinem et facilitatem
doctrinae, istud negotium in quatuor differentias seu capitula
separatur.

Quorum primum mathematica, quibus ad propositum indige-
mus, proponit; quod in tres partes dividitur. Quarum prima pro-
portionis definitiones, divisiones et ceteras proprietates ostendit.
Secunda, simili modo, de proportionalitate determinat. Tertia vero
quasdam suppositiones adiungit, ex quibus quasdam mathematicas
conclusiones demonstrat.

Capitulum autem secundum disputat quatuor opiniones, seu sec-
tas exortas de proportionem velocitatum in motibus; quod etiam,
secundum numerum opinionum illarum, in partes quatuor est
divisum.

Tertium capitulum veram sententiam de proportionem velocitatum
in motibus, in comparatione ad movementum et motorum potentias,
manifestat; quod etiam in duas partes est divisum. Quarum prima
quasdam conclusiones de proportionem velocitatum in motibus docet
et determinat. Secunda vero contra easdem obicit et solvit.

Capitulum autem quartum de proportionem velocitatum in motibus,
in comparatione ad moti et spatii pertransiti quantitates, pertractat,
et specialiter ad motum circulaarem descendit; quod in partes tres
similiter est partitum. Quarum prima quaedam mathematica,
ad illud necessaria, primo docet. Secunda vero quaedam opinio-
nes motorum et spatiorum pertransitorum, in comparatione ad magni-
tatem ostendit. Tertia autem circa proportionem elementorum
quasdam latentias manifestat.

Igitur ad propositum transeamus.

THOMAS OF BRADWARDINE'S TREATISE ON PROPORTIONS,
OR ON THE PROPORTIONS OF THE SPEEDS OF MOTIONS

INTRODUCTION

Since each successive motion is proportionable to another with
respect to speed, natural philosophy, which studies motion, ought
not to ignore the proportion of motions and their speeds, and, be-
cause an understanding of this is both necessary and extremely
difficult, nor has as yet been treated fully in any branch of philos-
ophy, we have accordingly composed the following work on the sub-
ject. Since, moreover (as Boethius points out in Book I of his *Arith-
metic*), it is agreed that whoever omits mathematical studies has
destroyed the whole of philosophic knowledge, we have commenced
by setting forth the mathematics needed for the task in hand, in
order to make the subject easier and more accessible to the stu-
dent. For the sake of this same ease and accessibility, the work
is also divided into four sections, or chapters.

The first of these, setting forth the necessary mathematics, is
subdivided into three parts, of which the first takes up the defini-
tions, types and other properties of proportion. The second deals
with proportionality in a similar fashion. The third adds certain
axioms, from which several mathematical conclusions are drawn.
Chapter two, on the other hand, argues against four opinions, or
schools of thought, which have arisen concerning the proportion
between the speeds of motions and, following the number of those
opinions, is divided into four parts.

Chapter three makes clear the correct understanding of the pro-
portion between the speeds of motions, with respect to both moving
and resisting powers, and this also is divided into two parts. The
first of these develops several theorems concerning the proportion
between the speeds of motions, and the second raises and settles
objections to them.

Chapter four treats of the proportion between the speeds of mo-
tions with respect to the quantities of the moved body and the inter-
val traversed, and includes a special discussion of circular motion.
It is divided into three parts, the first of which commences by
establishing the requisite mathematical material. Part two under-
takes the refutation of several opinions concerning the proportion
between the speeds of motions, with respect to the magnitudes both
of moved bodies and of intervals traversed, and sets forth the cor-
rect account. The third, finally, discloses certain hidden truths
concerning the proportions between the elements.
Let us then pass on to the task in hand.

Paulus Venetus: *Treatise on the first and the last instant*

Foreword

The instant of a thing can be twofold understood: inclusively and exclusively. An inclusive term < can be understood> with these two terms: “first being” and “last being”. An exclusive term < can be understood> with these two terms: “first not-being” and “last not-being”. A “first instant of being” is that instant in which it is true to say “the thing is now and immediately before that it was not”. A “last instant of not-being” is an instant in which it is true to say “the thing is now and immediately after that will not be”. A “first instant of no-being is that instant in which it is true to say “the thing is not now and immediately before that it was”. A “last instant of not-being” is an instant in which it is true to say “the thing is not and immediately after that it will be”.

About the difference between permanent and successive things

Note that there is a difference between a permanent and a successive thing. A permanent thing is <such a thing> which it does not contradict because of its nature to have all parts simultaneously, an example of that is the stone and other similar things. A successive thing is <such a thing> which it does contradict because of its nature to have all parts simultaneously; moreover, it belongs to its nature to have a first part and another posterior, so that when the first one is, the second one is not, and at the inverse. Examples of that are the days, the months and the hours. (...).

Anhang 2: ‚Tractatus de primo et ultimo instanti Pauli Veneti‘

⟨S⟩criptum Clarissimi doctoris Pauli Veneti feliciter incipit de primo et ultimo instanti 68^{va}
⟨Prohemium⟩

Instans⁵⁹ alicuius rei potest dupliciter intelligi: inclusive et exclusive. Inclusivum⁶⁰ extremum bis terminis ‚primum esse‘ et ‚ultimum esse‘. Exclusivum extremum bis terminis ‚primum non-esse‘ et ‚ultimum non-esse‘. ‚Primum instans esse‘ est instans in quo est verum dicere ‚res nunc est et immediate ante hoc non fuit‘. ‚Ultimum instans esse‘ est instans in quo est verum dicere ‚res nunc est et immediate post hoc non erit‘. ‚Primum instans non-esse‘ est illud in quo est verum dicere ‚res nunc non est et immediate ante hoc fuit‘. ‚Ultimum instans non-esse‘ est instans in quo est verum dicere ‚res nunc non est et immediate post hoc erit‘.

⟨De differentia inter permanentia et successiva⟩

Nota quod differentia est inter rem permanentem et rem successivam. Res permanens est cui non repugnat ex natura rei habere omnes partes simul, huius exemplum est lapis et similia. Res successiva est cui repugnat omnes partes habere simul, ymmo ex natura rei est habere unam partem priorem et aliam posteriorem, ita quod quando prior est, non est posterior et econtrario. Huius exemplum est dies, mensis et hora.

(...)

<3> For whatever thing, for which can be given a first instant of being, for the same <thing> cannot be given a last instant of not being. That can be proved since, if yes, let be *b* the first instant of being and *a* the last instant of not-being, I ask whether *a* and *b* are the same instant or different. They are not the same because the thing would be and not be in the same instant; therefore, <they are> different. I ask next, whether they are mediate or immediate. Not the latter, what is against the Philosopher in the Book VI of the *Physics*. If they are mediate, then, between *a* and *b* there is an interim period of time (*tempus medium*) in which it can neither be said that Socrates is nor that he not is, what is against a first principle. But, that in this time Socrates neither is nor not is, can be proved, since, if he was in this interim period of time, because of the fact that this time is before *b*, the thing would be before it was, or *b* would not be the first instant of being, what is against the hypothesis. If, on the contrary, he was not in that interim period of time, because of the fact that this interim period of time is after *a*, it follows that after *a* he would not be; therefore, *a* was not the last instant of not-being, what is against the hypothesis. But, if he was not in that interim period of time, then it would not be after his last <instant> of not-being, and so it there should be a last “posterior”, which is impossible.

<Propositiones>

35 <1> Si res⁶² habet esse solum per instans <tunc> est dare primum instans sui esse et ultimum instans sui esse. Ymmo idem est primum et ultimum instans sui esse. Ista propositio clara est.

<2> Rei permanentis per tempus durantis que in esse et <in> conservari dependet a re successiva non est dare primum instans nec ultimum. Ergo propositio vera, antecedens et consequentia⁶³ patebunt infrascriptis.

40 68^b <3> In quibuscumque rebus est dare primum instans esse, in eisdem non est dare ultimum instans⁶⁴ non-esse. Probatur | quia si sic sit *b* primum instans esse et *a* ultimum instans non-esse, quero aut *a* et *b* sunt idem instans aut diversa. Non idem quia in eodem instanti res esset et non esset, ergo diversa. Quero tunc aut mediata aut immediata. Non secundum quia tunc instantia essent immediata, quod est contra Philosophum 6^o *Physicorum*^a. Si mediata, ergo inter *a* et *b* 45 cadit tempus medium in quo non potest dici quod Sor est nec quod Sor non est, quod est contra primum principium. Sed quod in illo tempore <Sor> nec sit nec non sit, probatur, quia si in illo tempore medio esset, cum illud tempus sit ante *b*, res esset ante quam esset, vel *b* non esset primum instans esse, quod est contra ypotesim. Si autem in illo medio non esset, cum illud tempus medium

⁶¹ ante in] in scr. et del. F

⁶² res] correxi ex qua F

⁶³ ante consequentia] consequens scr. et del. F

⁶⁴ instans] correxi ex quod F

^a Aristoteles, *Physica* VI, 234a5–10.

50 sit post *a* sequitur quod post *a* non esset, ergo *a* non esset ultimum instans non esse, quod est contra ypotesim. Sed si in illo tempore medio non esset ergo post ultimum sui non-esse non esset, et sic esset dare posterius postremo, quod est impossibile.

55 <4> In quibuscumque rebus non est dare primum instans esse, in illis est dare ultimum instans non esse. Probatur sic: quilibet res habens esse de novo aut habet esse in instanti aut in tempore⁶⁵. Primum dari non potest ex secunda

THE
Science of Mechanics
in the Middle Ages

MARSHALL CLAGETT

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45 circumferentiam circuli faciente

23. Si recta finita super unum eius terminum quiescentem circula-
riter moveatur, omnes duas rectas terminatas ad punctum immotum
et alia puncta mota et velocitates istorum punctorum proportionales
certissime scias esse. Licet ista per primam conclusionem secunde
partis quarti capituli de proportionem velocitatum in motibus satis
apparet, potest tamen ut ibi breviter demonstrari

24. Quocunque motu locali signato potest motus localis uniformis
et continuus in omni proportionem recte finite ad rectam finitam velo-
cior et tardior inveniri. Unde manifestum est, quodcunque spacium
finitum quocunque tempore finito posse uniformiter et continue per-
transiri

26. Si quid continue localiter moveatur, in eodem instanti non
acquirere multos situs, nec in eodem situ in diversis instantibus esse
posse

45 circumferentiam *E* circumstantiam *T* /
faciente *T* facientem *E*
46 quiescentem *E* quiescente *T*
46-47 circulariter moveatur *T* circum-
moveatur *E*
47 rectas *om. T*
48 mota *T* immota *E* / proportionales
om. T
49 scias *T* habere *E* / ista *E* prima *T* /

primam *T* proximam *E* / conclusionem
E conclusionis *T*
50 quarti *T* aut *E* / satis *om. E*
55 quocunque *E* quodcunque *T* / finito
T finitum *E*
55-56 pertransire *E*
57 continue *om. E*
58 aquirat *T* / nec *om. E*

Document 4.4

William Heytesbury, *Rules for Solving Sophisms**

[Part VI. Local Motion]

[Prologue]

THERE are three categories or generic ways in which motion, in the strict sense, can occur. For whatever is moved, is changed either in its place, or in its quantity, or in its quality. And since, in general, any successive motion whatever is fast or slow, and since no single method of determining velocity is applicable in the same sense to all three kinds of motion, it will be suitable to show how any change of this sort may be distinguished from another change of its own kind, with respect to speed or slowness. And because local motion is prior in nature to the other kinds, as the primary kind, we will carry out our intention in this section, with respect to local motion, before treating of the other kinds.

[1. Measure of Uniform Velocity]

Although change of place is of diverse kinds, and is varied according to several essential as well as accidental differences, yet it will suffice for our purposes to distinguish uniform motion from nonuniform motion. Of local motions, then, that motion is called uniform in which an equal distance is continuously traversed with equal velocity in an equal part of time. Nonuniform motion can, on the other hand, be varied in an infinite number of ways, both with respect to the magnitude, and with respect to the time.

In uniform motion, then, the velocity of a magnitude as a whole is in all cases measured (*metietur*) by the linear path traversed by the point which is in most rapid motion, if there is such a point. And according as the position of this point is changed uniformly or nonuniformly, the complete motion of the whole body is said to be uniform or difform (non-uniform). Thus, given a magnitude whose most rapidly moving point is

* The translation kindly supplied to me by Ernest Moody has been slightly altered.

moved uniformly, then, however much the remaining points may be moving nonuniformly, that magnitude as a whole is said to be in uniform movement . . .

[2. Measure of Nonuniform Velocity]

In nonuniform motion, however, the velocity at any given instant will be measured (*attenditur*) by the path which *would* be described by the most rapidly moving point if, in a period of time, it were moved uniformly at the same degree of velocity (*uniformiter illo gradu velocitatis*) with which it is moved in that given instant, whatever [instant] be assigned. For suppose that the point *A* will be continuously accelerated throughout an hour. It is not then necessary that, in any instant of that hour as a whole, its velocity be measured by the line which that point describes in that hour. For it is not required, in order that any two points or any other two moving things be moved at equal velocity, that they should traverse equal spaces in an equal time; but it is possible that they traverse unequal spaces, in whatever proportion you may please. For suppose that point *A* is moved continuously and uniformly at *C* degrees of velocity, for an hour, and that it traverses a distance of a foot. And suppose that point *B* commences to move, from rest, and in the first half of that hour accelerates its velocity to *C* degrees, while in the second half hour it decelerates from this velocity to rest. It is then found that at the middle instant of the whole hour point *B* will be moving at *C* degrees of velocity, and will fully equal the velocity of the point *A*. And yet, at the middle instant of that hour, *B* will not have traversed as long a line as *A*, other things being equal. In similar manner, the point *B*, traversing a finite line as small as you please, can be accelerated in its motion beyond any limit; for, in the first proportional part of that time, it may have a certain velocity, and in the second proportional part, twice that velocity, and in the third proportional part, four times that velocity, and so on without limit.

From this it clearly follows, that such a nonuniform or instantaneous velocity (*velocitas instantanea*) is not measured by the distance traversed, but by the distance which *would* be traversed by such a point, *if* it were moved uniformly over such or such a period of time at that degree of velocity with which it is moved in that assigned instant.

[3. Measure of Uniform Acceleration]

With regard to the acceleration (*intensio*) and deceleration (*remissio*) of local motion, however, it is to be noted that there are two ways in

which a motion may be accelerated or decelerated: namely, uniformly, or nonuniformly. For any motion whatever is *uniformly accelerated* (*uniformiter intenditur*) if, in each of any equal parts of the time whatsoever, it acquires an equal increment (*latitudo*) of velocity. And such a motion is uniformly decelerated if, in each of any equal parts of the time, it loses an equal increment of velocity. But a motion is *nonuniformly accelerated or decelerated*, when it acquires or loses a greater increment of velocity in one part of the time than in another equal part.

In view of this, it is sufficiently apparent that when the latitude of motion or velocity is infinite, it is impossible for any body to acquire that latitude uniformly, in any finite time. And since any degree of velocity whatsoever differs by a finite amount from zero velocity, or from the privative limit of the intensive scale, which is rest—therefore any mobile body may be uniformly accelerated from rest to any assigned degree of velocity; and likewise, it may be decelerated uniformly from any assigned velocity, to rest. And, in general, both kinds of change may take place uniformly, from any degree of velocity to any other degree.

COMMENTARY

Notice in this passage the definitions of uniform velocity, uniform acceleration, and instantaneous velocity. In the definition of uniform velocity, Heytesbury speaks of the traversal of an equal space in an equal part of the time. Thus he failed to say in *any* equal parts of the time. That this would appear to be understood by him is clear, however, from his definition of uniform acceleration in terms of the acquisition of an equal increment of speed in *any* equal parts of the time. Heytesbury's contemporary, Richard Swineshead, was careful to specify that uniform velocity is to be defined by the traversal of an equal distance in *every* (*omni*) equal period of time (see the succeeding document of Swineshead). Hence Swineshead, at least, was anticipating Galileo's admonition to include the word "any" in a proper definition of uniform motion (see the Galileo selection, Doc. 4.7, below).

One final important point should be noticed about this selection from Heytesbury's *De motu*. For him instantaneous velocity is to be measured or determined by the path which *would* be described by a point if that point were to move during some time interval with a uniform motion of the velocity possessed at the instant. It will be noticed in the passage from Galileo's *Two New Sciences* quoted below as Document 4.7 that he did much the same thing as Heytesbury.

*Regule solvendi sophismata Guillelmi Heytesberi**

Tria sunt predicamenta vel genera in quorum quolibet contingit proprie motum esse; mutatur enim localiter, quantitative, aut qualitative, quodlibet quod movetur. Et cum universaliter motus quilibet successivus velox sit vel tardus, nec aliquid est idem univocum penes quid attendi poterit velocitas in hiis tribus, conveniens erit ostendere qualiter quecumque mutatio huiusmodi, quo ad eius velocitatem seu tarditatem, ab alia sui generis distinguatur. Et quia motus localis naturaliter precedit alios tanquam primus, circa ipsum in hac parte transcurrens saltem intentio ceteris premitratur.

Loci autem mutatio, quamvis diversas habeat species, et tam essentialibus quam etiam accidentalibus differentiis pluribus varietur, ad propositum tamen sufficiet motum uniformem distinguere a difformi. Motuum igitur localium dicitur uniformis quo equali velocitate continue in equali parte temporis spacium pertransitur equale. Difformis quidem in infinitum variari potest, et respectu magnitudinis et etiam quo ad tempus.

In uniformi itaque, penes lineam a puncto velocissime moto descriptam, si quis huiusmodi fuerit, quanta sit totius magnitudinis mote velocitas universaliter metietur. Et penes hoc quod punctus talis uniformiter seu difformiter mutat situm, totius totus motus uniformis

* MS Bruges, Stadsbibliotheek 497, f. 56r (A); MS Bruges, Stadsbibliotheek 500, ff. 56v-57r (B); MS Vat. lat. 2136, ff. 24v-25r (V); Edition of Venice, 1494, ff. 37r-39v (Ed). Some differences in orthography between these various copies

- 2 quorum quolibet: quibus *Ed*
 3 proprie... esse: motum fieri proprium *om. Ed. ille dicitur V / quo BV que A est quo Ed*
 4 *Ed* / mutatur: movetur *B* / qualitative
 5 aut quantitative *A* / aut: vel *B*
 6 universaliter *BVEd* uniformiter *A*
 7 idem *om. AV*
 8 erit: est *Ed*
 9 huiusmodi: prius in *A* / seu: seu eius *A*
 10 ab: ab ad *B* / alia: altera *A*
 11 etiam *om. Ed*
 12 sufficit motum *tr. B*
 13 igitur: ergo *Ed* / motuum... localium
 14 *AEd* motus igitur localis *B* / dicitur *om. Ed*

dicitur vel difformis. Unde data magnitudine cuius punctus velocissimus uniformiter moveatur, quantumcumque difformiter residua omnia differantur, uniformiter moveri conceditur tota proposita magnitudo.

- 23 Posito nempe casu quo mote magnitudinis nullus sit punctus velocissime motus, penes lineam quam describeret punctus quidam qui indivisibiliter velocius moveretur, quam aliquis in magnitudine illa data tota, totius velocitas attendetur: Sicut posito quod continue incipiant corrumpi puncta extrema, aut quod nulla sint ultima puncta illius sicut accidit in linea girativa que ponitur infinita. Movetur enim omnis magnitudo, mota localiter, ita velociter sicut aliqua pars ipsius, aut sicut aliquis eius punctus; unde in casu isto indivisibiliter velocius movetur talis magnitudo quam aliquis eius punctus. Et ideo, iuxta illud, conceditur tanquam possibile quod continue tardius et tardius movebitur mobile *A* per horam, et tamen continue per eandem horam erit ita quod quilibet punctus illius qui movetur, velocius movetur quam prius, et intendit motum suum. Et similiter quod *B* magnitudo per horam continue eque velociter movebitur, et uniformiter saltem quo ad tempus, et tamen quilibet punctus illius continue per eandem horam tardabit motum suum. Primum, per continuam corruptionem punctorum extremorum ipsius mobilis, satis poterit verificari; et secundum absque inconvenienti concedi poterit de linea girativa. Potest etiam concedi tanquam imaginabile similiter, quod *A* magnitudo per horam continue velocius et velocius movebitur, et tamen continue erit ita per eandem horam, quod quilibet punctus motus ipsius *A*
- 31 illius: istius *A* / accidet *Ed* / enim: autem *A*
 32 mota *om. B* / pars ipsius: eius pars *Ed*
 33 isto *AB* illo *VEd*
 34 eius punctus *tr. BV*
 35 illud: idem *A*
 36 mobile *A tr. VEd*
 37 illius *BV* istius *A* eius *Ed* / qui *om. B* / movetur *tr. B post prius*
 38 ante quod *add. Ed* / conceditur
 39 illius: eius *Ed*
 40 ipsius *V/B* illius *Ed* istius *A*
 41 etiam concedi *tr. Ed*
 42 puncta *B* / motus ipsius *A BV* istius *A* illius *Ed*
- 22 post difformis *tr. A* dicitur
 23 ante uniformiter *add. A* motus / residua omnia *tr. Ed* reliqua omnia *V* residua *A*
 24 conceditur: contra *B*
 24-25 proposita magnitudo *tr. VEd*
 26 mote: note *Ed* / magnitudinis motus *A*
 27 penes: tunc penes *V* / describeret: difformis et *B*
 28 movetur *A* / ante quam *add. Ed* ac si esset
 29 illa *om. B* / totius velocitas: tota velocitas totius *A* / attendatur *Ed*
 30 incipiat *B* / puncta *om. Ed* / ultima puncta *tr. V*

[gradum] sicut per illum motum localem uniformiter difformem. Unde quiscunque motus difformis, et etiam quecunque alia qualitas, alicui gradui correspondet....

Document 4.6

John of Holland,* *On Motion*

.. OF local motions some are uniform, some are difform (nonuniform = *difformis*). Uniform local motion is of two kinds. Some is uniform as to magnitude and some is uniform as to time. Local motion uniform as to magnitude is the motion of some magnitude whereby the whole magnitude is moved equally fast (*equevelociter*) as any part at all and so the movement of a stone downward is moved uniformly as to magnitude. Motion uniform as to time is the motion of some moving body (*mobilis*) whereby the body traverses an equal space in every equal part of the time (*in omni parte equali temporis*) in which it is moved. From this it follows that the ninth sphere is moved uniformly as to time, but nonuniformly as to magnitude, because the parts of the ninth sphere nearer to the equinoctial circle are moved more quickly than the parts nearer the poles.

2. Difform motion is of two kinds. Some is difform as to magnitude and some is difform as to time. Motion difform as to magnitude is the motion of a magnitude where some part of the magnitude is moved more slowly than the whole magnitude (*actually*, than another part), as in the case of the movement of the ninth sphere. Similarly the motion of any sphere at all when rotated is difform as to magnitude.

3. Movement difform as to time is the motion of a mobile body whereby the body traverses more space in one part of the time than in some other equal part of the time. Motion difform as to time is twofold: some uniformly difform, some difformly difform. Local motion uniformly difform is described by the [English] Calculators as follows: "Local motion uniformly difform is a difform movement in *any* designated part of which the middle speed of that part exceeds the minimum terminal speed by the same increment (*latitudo*) as the middle is exceeded by the maximum terminal speed." Motion difformly difform occurs when uniformly difform movement is nonexistent.

* Another of John's works, his *Libri de instanti*, is dated 1369 at Prague in MS Oxford, Bodl., Canon. Misc. 177, f. 61v.

4. *To be moved as to time* is twofold, either uniformly or difformly. *To be moved uniformly as to time* is to traverse an equal space in every equal period of time. . . . *To be moved difformly as to time* is twofold, to increase speed (*intendere motum*) or to decrease speed (*remittere motum*). *To increase speed* is twofold, uniformly or difformly. *To increase speed uniformly* is to acquire an equal increment of speed (*latitudo motus*) in every equal part of the time. . . .

COMMENTARY

My principal purpose in including this passage from John of Holland's *De motu* is to show how the English ideas spread quickly to other parts of Europe. This document reveals their presence at Prague in the 1360's. Succeeding chapters will show how they spread to France and Italy by about 1350.

Worth special notice is John's quotation of a common English definition of uniform acceleration, namely that in any designated part of the time of that accelerated movement, $V_f - V_m = V_m - V_i$, where V_f , V_m , V_i are respectively the final, middle, and initial instantaneous velocities of the designated time period.

Finally, it is of some interest to observe that John of Holland called his Merton College predecessors the "Calculators." This was a title that was to stick, particularly to Richard Swineshead, who was known in Italy and elsewhere in the last part of the fourteenth and throughout the fifteenth century as the Calculator. The whole technique of treating qualities and motion in the Merton manner became known as "the calculations."

*De motu Johannis de Hollandia**

Motuum localium quidam est uniformis, quidam est difformis.

Motus localis uniformis est duplex; nam quidam est uniformis quo ad magnitudinem et quidam est uniformis quo ad tempus. Motus

localis uniformis quo ad magnitudinem est motus alicuius magnitudinis quo ipsa magnitudo movetur equevelociter cum qualibet parte sua, et sic lapis motus deorsum movetur uniformiter quo ad magnitudinem. Motus uniformis quo ad tempus est motus alicuius mobilis

* MS Oxford, Bodl. Canon. Misc. 177, Marco Lat. VIII, 19, f. 1r. (V); MS Vat. f. 100v, (B); MS Venice, Bibl. Naz. San lat. 1108, f. 144r (C).

2 quidam²: et quidam C / est² om. VC 6-7 parte sua tr. BC

5 uniformis tr. B post magnitudinem 7 movetur om. B

quo ipsum mobile in omni parte equali temporis pro quo illud movetur pertransit spatium equale.

Ex isto sequitur quod nona spera uniformiter movetur quo ad tempus, difformiter quo ad magnitudinem, quia partes nonne spere circulo equinoctiali propinquiores velocius moventur partibus existentibus circa polos.

Motus difformis est duplex; nam quidam est difformis quo ad magnitudinem et quidam quo ad tempus. Motus difformis quo ad magnitudinem est motus magnitudinis quo movetur eiusdem magnitudinis aliqua pars tardius ipsa tota magnitudine (i.e., alia parte magnitudinis) mota, ut motus nonne spere; et similiter motus cuiuslibet spere mote circulariter est difformis quo ad magnitudinem.

Motus difformis quo ad tempus est motus mobilis quo ipsum mobile in una parte temporis plus pertransit quam in alia parte temporis sibi equali. Motus difformis quo ad tempus est duplex; nam quidam est uniformiter difformis, quidam difformiter difformis.

Motus localis uniformiter difformis describitur a calculatoribus sic: Motus localis uniformiter difformis est motus difformis, cuius quacunque parte signata medius gradus illius partis per equalem latitudinem excedit extremum remissius eiusdem sicut ipse ab extremo intensiori illius partis exceditur.

Motus difformiter difformis est motus difformis non existens uniformiter difformis. Moveri quo ad tempus est duplex, vel uniformiter vel difformiter. Uniformiter moveri quo ad tempus est in omni parte temporis equali equale spatium pertransire, vel secundum rem vel secundum equevalentiam. Difformiter moveri quo ad tempus est duplex, vel est intendere motum vel est remittere motum. Intendere motum est duplex, vel uniformiter intendere vel difformiter intendere. Uniformiter intendere motum est in omni parte temporis equali equalem latitudinem motus acquirere. Verbi gratia, quiescat Sortes

9 equali om. VC / pro quo VB quod C 19 ut V et B / motus tr. C post circulariter in linea 20

10 pertransit spatium tr. C 22 plus VC non B / quam VC tantum

11 quod om. C / uniformiter movetur tr. B

12 et difformiter C 23 difformis om. V

13 propinquius V 24 quidam² V et quidam BC

16 post quidam add. C est difformis / 25 sic tr. C ante calculatoribus

difformis om. C 28 remissius om. V

17 magnitudinis VC difformis magnitudinis B 29 exceditur tr. C ante ab in linea 28

18 ipsa V ipsa scilicet B movetur ipsa C 33 pertransire BC pertransiri V

38 equalem VC equalis B

Document 5.2

Proofs of Propositions Posited in the Rules for Solving Sophisms Attributed to William Heytesbury

EVERY increment of velocity (*latitudo motus*) uniformly acquired or lost will correspond to its mean degree [of velocity]. This means that a moving body uniformly acquiring or losing that increment will traverse in some given time a magnitude completely equal to that which it would traverse if it were moving continuously through the same time with the mean degree [of velocity].

Proof. I assume a total [velocity] increment from zero degree to a degree of eight. And I take three bodies, namely, a , b , c , all now moving with the mean degree of that velocity increment (i.e., with a velocity of 4).... Let a move through an hour uniformly at that degree. Let b accelerate uniformly from that degree to a degree of eight in half of the hour and let c decelerate uniformly from that mean degree to zero degree in the same half hour. Furthermore, I take a body d which uniformly acquires the whole designated increment (i.e., from 0 to 8) in the whole hour. Then it is argued as follows.

Body d will traverse just as much space in the whole hour as do b and c in one-half of the hour. But b and c will traverse in the half hour just as much as a will traverse in the whole hour. Therefore d will traverse just as much in the whole hour as a . The [syllogistic] consequence is evident.¹⁵ From the consequent it is [further argued] as follows: d will traverse in the whole hour as much as a in the same hour. But a will traverse neither more nor less in the whole hour than that which moves with the mean

¹⁵ For fourteenth century logicians, syllogisms were only one species of the generic, logical form called a consequence, and thus this author's arguments are syllogistic consequences. See P. Bochner, *Medieval Logic* (Chicago, 1952), pp. 52-53; E. A. Moody, *Truth and Consequence in Medieval Logic* (Amsterdam, 1953), p. 61; and I. M. Bocheński, *Formale Logik* (Munich, 1956), pp. 219-43.

degree of the increment. Therefore d will traverse, in acquiring the velocity increment, just as much space as if it were continually moved with the mean degree of the increment. The inference is obvious, and the major is argued.

Body d will traverse as much in the first half of the hour as c in the same half. But d will traverse in the second half of the hour as much as b in the first. Therefore d will traverse so much in the whole hour as b and c joined together in half of that hour. The consequence is obvious and also the major premise. But that a will traverse in the whole hour as much as b and c in half of the hour, which is the minor premise of the argument [of the first syllogism], is argued thus. If b were not increased and c were not decreased, they would, joined together, in one-half hour traverse as much as a in the whole hour. But b and c are as much now as they were then. Therefore, b and c will traverse in half an hour as much as a in the whole hour. The consequence is obvious; and the minor premise is argued.

In the half hour a will traverse more space than c by the same amount that b traverses more space than a in the half hour. Therefore, in comparing the excess of a over c , or b over a , in that half hour, both of them, namely, b and c , taken together would traverse the same amount as if neither b nor c were increased or diminished. And therefore b and c will traverse the same amount now as then. Therefore, etc. And just as it is argued concerning moving body d , so it can be argued for any moving body acquiring that increment in an hour; therefore, etc.

If any moving body uniformly increases its motion from zero degree up to any degree in an hour, it will traverse in the second half hour precisely three times the distance that it will traverse in the first half of the same [hour].

Proof: The degree to which corresponds the whole motion made in the first part of the hour is one-third the degree to which the whole motion completed or made in the second half hour corresponds. Therefore the distance traversed in the first half is one-third the distance traversed in the second half hour. The consequence is proved. Just as the degree to which corresponds the motion in the first half hour is related to the degree to which corresponds the motion in the second half, so is related the distance traversed in the first part of the hour to the distance traversed in the second. But from this—that the degree to which corresponds the motion in the first half hour is one-third the degree to which corresponds the motion in the second half hour—it follows that the distance traversed [in the first half will be one-third that traversed in the second half].

And the minor is argued thus. The degree to which corresponds the motion completed in the first half hour is one-third, etc., since I posit that the moving body *a* uniformly increases its motion in an hour from zero degree up to eight. And then it is argued thus: a degree of two is one-third of a degree of six. But to a degree of two corresponds the motion in the first half, and to a degree of six corresponds the motion in the second half. Therefore, etc. The minor is proved: the whole motion made in the first half hour corresponds to its mean degree, which is two. And the whole motion made in the second half hour corresponds to its mean degree, which is six, as is obvious to one who attends closely. Therefore, etc.

COMMENTARY

This selection consists of two theorems—the mean velocity theorem and the theorem comparing the distances traversed in the first and second half of the time by a body uniformly accelerating. It will be noticed that the form of the arguments in the proof of the first theorem is completely syllogistic (see note 15). Before analyzing the syllogisms given, we should notice the data of the proof:

1. A body *a* moves uniformly through an hour with a velocity of 4.
2. A body *b* accelerates uniformly from 4 to 8 in one-half hour.
3. A body *c* decelerates uniformly from 4 to 0 in one-half hour.
4. A body *d* accelerates uniformly from 0 to 8 in the whole hour.
5. Bodies *a*, *b*, *c*, and *d* move respectively through distances S_a , S_b , S_c , and S_d .

Then Heytesbury states the first syllogism.

Syllogism I

If (1) $S_d = S_b + S_c$
and (2) $S_b + S_c = S_a$
then (3) $S_d = S_a$

He follows this by the statement of an obvious syllogism not requiring demonstration (S_{am} in (4) being the distance traversed at mean speed of the increment acquired by *d*):

Syllogism II

If (3) $S_d = S_a$
and (4) $S_{am} = S_a$
then (5) $S_d = S_{am}$

Having outlined the principal syllogisms, he then goes on to demonstrate

the veracity of the major and the minor premises, (1) and (2), of Syllogism I. The proof of the major is rather obvious, since *b* and *c* are precisely equivalent to the two halves of *d*. The minor premise [(1) (2)] is the crux of the whole demonstration. This form of the proof is as follows (S_b , S_c being the distances traversed if *b* and *c* were held at the mean degree rather than accelerating and decelerating):

Since (6) $S_b + S_c = S_a$
and (7) $S_b + S_c = S_b + S_c$
therefore (8) $S_b + S_c = S_a$

The crucial part of this supplementary syllogism is the minor premise (7). This is never really proved. It is simply intuitively grasped that the $S_a/2$ covered in the first half (or its equivalent S_b) exceeds S_c by the same amount that $S_a/2$ traversed in the second half (S_c) is exceeded by S_b . Now with both (1) and (2) assumed to be proved, (3) follows; and from (3) and the defined fact of (4) follows the mean speed theorem, at least as applied to bodies accelerating from rest. A succeeding proof, not included in our translation here, generalizes the law for all bodies uniformly accelerating from any given velocity. The reader may examine proofs of this more generalized theorem by Swineshead and Dumbleton in the two succeeding documents.

The three-to-one distance theorem is proved immediately from the mean velocity theorem. Its proof needs no additional commentary.

*Probationes conclusionum tractatus regularum
solvendi sophismata Guillelmi Heytesberi**

Omnis latitudo motus uniformiter acquisita vel deperdita correspondebit gradui medio ipsius, i.e., quod mobile idem ipsam latitudinem uniformiter acquirens seu deperdens in aliquo tempore dato equalem omnino magnitudinem petransibit ac si ipsum continue per equale tempus moveretur medio gradu. Istud probatur sic: Accipio totam latitudinem a non gradu ad gradum ut octo. Et capio tria mobilia, scilicet, *a*, *b*, *c*, sub medio gradu illius latitudinis. Quorum *a* ponatur petransire [4?] pedale in hora. Et moveatur per horam illo

* Edition of Venice, 1494, ff. 198v–199r (*Ed*); cf. MS Venice, Bibl. Naz. San Marco Lat. VI, 71, f. 129v (*V*). The manuscript is quite defective and it proved of no profit to collate the two. Cf. Oxford, Bodl. Canon. Misc. 376, ff. 23–32 (contains only chapters 3 to 5, and hence not the relevant part).

gradu. Et intendat b uniformiter motum suum ab isto gradu usquam ad gradum ut octo in medietate illius hore. Et remittat c uniformiter motum suum ab isto medio gradu ad non gradum in eadem medietate hore. Et capio d mobile quod uniformiter acquirat totam latitudinem iam assignatam in tota ista hora. Tunc arguitur sic:

Tantum pertransibit d in tota hora quantum pertransibunt b , c in medietate hore. Sed tantum pertransibunt b , c in medietate hore quantum pertransibit a in tota hora. Igitur tantum pertransibit d in tota hora quantum pertransibit a in tota hora. Consequentia patet et ex consequenti arguitur sic: tantum pertransibit d in tota hora quantum pertransibit a in eadem hora. Sed nec maius nec minus pertransibit a in tota hora quam a movendo medio gradu illius latitudinis. Igitur tantum pertransibit sic acquirendo istam latitudinem ac si pertransiret continue movendo medio gradu illius latitudinis d . Consequentia patet et maior arguitur.

Tantum pertransibit d in prima medietate hore quantum c in eadem medietate hore. Sed tantum pertransibit d in secunda medietate quantum b in prima. Igitur d pertransibit tantum in tota hora quantum b , c coniuncti in medietate illius hore. Consequentia patet et maior similiter. Sed quod tantum pertransibit a in tota hora quantum pertransibunt b , c in medietate hore que est minor illius argumenti arguitur sic. Si nec b intenderetur nec c remitteretur pertransirent coniuncti in medietate hore tantum quantum a in tota hora. Sed tantum pertransibunt b , c nunc sicut tunc. Igitur tantum pertransibunt b et c in medietate hore quantum a in tota hora. Consequentia patet et minor arguitur.

Quia quanto minus pertransiretur a c quam ab a in medietate hore tanto plus pertransiretur a b quam ab a in eadem medietate. Igitur comparando excessum quo pertransiretur plus ab a quam a c vel a b quam ab a in illa medietate hore ab utroque, scilicet b , c equaliter pertransiretur coniunctim a b , c si nullum illorum intenderetur nec aliquid remitteretur, et ultra igitur tantum pertransibit b , c nunc quantum tunc. Igitur, etc. Et sic arguitur de d mobili sic potest argui de quocumque mobili uniformiter acquirendo istam latitudinem in hora. Igitur, etc....

Si aliquid mobile uniformiter intendat motum suum a non gradu usque ad aliquem gradum in hora triplum precise pertransibit in secunda medietate hore ad hoc quod pertransibit in prima medietate eiusdem. Probatur: nam gradus cui correspondet totus factus motus in

prima parta hore est subtripulus ad gradum cui correspondet totus motus finitus vel factus in secunda medietate hore. Igitur pertransitum in prima medietate est subtripulum ad pertransitum in secunda medietate hore. Consequentia probatur: Quia sicut se habet gradus cui correspondet motus in prima medietate hore ad gradum cui correspondet motus in secunda medietate sic se habet pertransitum in prima parte hore ad pertransitum in secunda. Sed ex quo gradus cui correspondet motus in prima medietate hore est subtripulus ad gradum cui correspondet motus in secunda medietate hore sequitur quod erit pertransitum, etc.

Et minor arguitur sic: quod gradus cui correspondet motus factus in prima medietate hore est subtripulus etc. quia posito quod a mobile uniformiter intendat motum in hora a non gradu usque ad octo: Et tunc arguitur sic: gradus ut duo est subtripulus ad gradum ut vi (6) sed gradui ut duo correspondet motus in prima medietate et gradui ut vi correspondet motus in secunda medietate. Igitur, etc. Minor probatur: totus motus factus a prima medietate hore correspondet medio suo gradui qui est ut duo. Et totus motus factus in secunda medietate hore correspondet medio suo gradui qui est ut vi ut patet intuenti. Igitur, etc.

Document 6.4

Blasius of Parma, *Questions on the Treatise on the Latitudes of Forms*

1. QUESTION III. Whether every uniformly difform latitude corresponds to its mean [This] is obvious from all those who commonly speak on this matter, as well as from rational exposition. It is argued as follows: I posit that *A* decelerates (*remittat motum suum*) uniformly from *C* degree [of velocity]—e.g., 4—to zero degree during half of the hour, while *B* accelerates (*intendat motum suum*) uniformly in the same half hour from exactly *C* degree [of velocity] to a degree which is twice *C*. With this posited, it follows that *A* and *B* [together] will traverse exactly as much [space], as they would traverse if they were moved continually with *C* degree [of velocity]. This is obvious, for just as much as *B* acquires of velocity increment (*latitudo motus*) in accelerating, that much exactly of velocity increment does *A* lose in decelerating. Therefore, *B* by its acceleration will traverse as much more [space] than if moved uniformly with *C* velocity] as *A* by its deceleration traverses less [space] than if moved

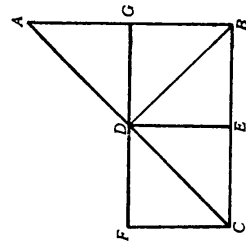


Fig. 6.12

uniformly with *C* velocity]. Therefore, exactly as much [space] will be traversed by *A* and *B* [together] as if they were moved continually with *C* degree of velocity [*gradus velocitatis*].

2. Furthermore, if *C* were a body uniformly difformly hot, whose latitude, for example, would be represented by triangle *ABC* (see Fig. 6.12),

and in whose more intense extreme were applied one corrupting [agent] and in whose more remiss extreme its contrary, in the same proportion, so that the corrupting [agent] would corrupt as much of latitude *ABC* in the more intense extreme as its contrary [agent] would introduce in the more remiss extreme, then it is evident that at the end of the action, latitude *ABC* would be uniform, and it would be of precisely the same [quantity] of intension as it was before the action of these [agents] . . .

3. . . . I posit some suppositions: 1. Latitudes of forms are represented (*presentantur*) by geometrical figures. The author premises this supposition. 2. The line which divides two sides of any triangle into equal parts is one-half the third side. This is obvious from the fourth [proposition] of the sixth [book] of Euclid. Let there be a triangle *ABC* (see Fig. 6.12), and the line cutting two of the sides into equal parts we let be *DE*. I say that the line *DE* is one-half the line *AB*. For $BC/BE = AC/AD = AB/DE$, and $BC = 2BE$. Therefore $AB = 2DE$, and $DE = 1/2 AB$. 3. Any parallelogram with one side equal to the mean degree and the other equal to the line of extension (*extensio*) is equal to the triangle by which a latitude uniformly difform is representable to us and whose mean degree line is the small side of the given parallelogram. This is apparent, [for], with triangle *ABC* given, whose mean line is *DE*, a parallelogram is formed of the aforementioned lines, and it is *BGCF*. This supposition is evident by the facts of geometry . . .

The second conclusion: every uniformly difform latitude, uniformly difformly acquired, whether it is a latitude of something with a "permanent" *esse* [like a quality] or is a latitude of something with a "successive" *esse* [like velocity], corresponds to its mean degree. This is clear by the first and third suppositions.

COMMENTARY

1. The first "proof" or explanation of the mean speed theorem given by Blasius is obviously very much like those found in the treatises of Heytesbury (or rather the author of the *Probationes* whom we have assumed to be Heytesbury) and Swineshead. I think there is little doubt that Blasius drew his proof from the English tradition. It is of the Merton *genre* that emphasizes the symmetrical gain and loss of velocity and space traversed in comparing uniformly accelerating and decelerating bodies with one moving uniformly at the mean velocity.

2. The second example, which concerns the action of two contrary agents which act so as to reduce a uniformly difform quality to uniformity,

is drawn directly from Giovanni di Casali's *De velocitate motus alterationis* (see Latin text of Doc. 6.2, pp. 389-90, lines 66-99), except that Casali converts a uniform quality into one that is uniformly difform by the continuous application of two contrary agents; in short Blasius' procedure is the reverse of Casali's. But it is clear that Blasius has used Casali's treatise as a source.

3. The ultimate source of Blasius' geometric proof of the mean speed theorem is either Casali's treatise or, more probably, the *De configurationibus qualitatum* of Oresme. Obviously, Blasius did not get the proof from the *De latitudinibus formarum*, since that treatise has no statement or proof of the mean speed theorem. Like the proof in the *De configurationibus* (but unlike that of Casali), Blasius appears to represent his intensity lines by vertical rather than horizontal perpendiculars. Needless to say, however, Blasius uses "latitude" in the sense labeled as "incorrect" by Oresme, i.e., as a name for the whole quantity of the quality or for the quantity of velocity. He obviously gets this usage from the *De latitudinibus formarum*, which served as the point of departure for his *Questions*.

Questions super tractatu de latitudinibus formarum

*Blasii de Parma**

Questio III. Utrum quilibet latitudo uniformiter difformis correspondeat suo gradui medio . . . patet per omnes communiter loquentes in hac materia, et etiam ratione. Arguitur sic: et volo quod *A* remittat motum suum uniformiter a *C* gradu, ut 4, in medietate huius hore, usque ad non gradum; et *B* intendat motum suum uniformiter, in eadem medietate illius, a *C* gradu precise ad gradum duplum ad *C*. Quo posito, sequitur quod *A* et *B* precise tantum spadi-

* Edition of Venice, 1505 (*Ed*); no pag. case letters for figures again changed to but ff. 305-322; this quest. ff. 311-322). capital letters. Additional manuscripts not Cf. MS Oxford, Bodl. Canon. Misc. 177, used here are Bodl. Canon. Misc. 181, ff. 97v-113v (*B*); old pag.; new pag. ff. 115 c., ff. 64-66, Milan, Bibl. Ambros. F. 110v-113v; this quest. old pag. 99r-100v; new pag. 112r-113v); and cf. MS Venice, Bibl. Naz. San Marco Lat. VI, 155, ff. 88r-92 (*V*); this quest. ff. 90r-92r). Lower-

3 Questio III *Ed. om. V* quarto queritur *B* / uniformiter difformis *VEd om. B*
 5 in *VB* de *Ed* / et etiam *Ed. om. V* et *B*
 7-8 uniformiter *BV om. Ed*
 8 medietate *VEd* hora usque *B* / illius *V om. BEd*
 9 *B* correct ex *C* in *MSS* et *Ed*

10 cium pertransibunt, quantum pertransirent si continue moverentur *C* gradu. Quod patet, nam quantumcunque *B* acquirit de latitudine motus per suam intensionem, tantum precise deperdet *A* de latitudine per remissionem sui motus. Ergo quanto magis per intensionem sui motus *B* pertransibit, tanto minus *A* per remissionem sui motus pertransibit. Ergo, tantum precise erit pertransitum ab *A* et *B* ac si continue moverentur *C* gradu velocitatis.

15 Praeterea si *C* esset unum corpus uniformiter difformiter calidum cuius latitudo, gratia exempli, presentaretur per triangulum *ABC*, (*vide* Fig. 6.12) et in extremo eius intensiori applicaretur unum cor-
 20 rumpens et in extremo remissiori suum contrarium corrumpet de latitudine *ABC* in extremo intensiori, tantum suum contrarium in extremo remissiori introduceret de latitudine, tunc patet quod in fine actionis latitudo *ABC* esset uniformis, et precise esset tante inten-
 25 sionis quante erat ante actionem istorum.

In hac questione erunt quatuor articuli. In quorum primo evidentie premitte de sunt, in secundo distinctiones, in tertio conclusiones de quesito, in quarto difficultates. Quantum ad primum nota quod latitudo illa est uniformiter difformis cuius medius gradus per tantam latitudinem excedit non gradum sicut per quantum latitudinem ipse medius gradus exceditur ab intensiori gradu eiusdem latitudinis. Nota secundo quod non idem est sermo positus de latitudine uniformiter difformi motus localis et caliditatis, quoniam latitudo motus localis non habet esse permansivum, sed bene latitudo caliditatis. Et pro isto

10 pertransibunt *BV* transibunt *Ed* /
 11 pertransient *BV om. Ed*
 11 B *VEd A B*
 12 intensionem *BEd* intensiorem *V* /
 13 deperdet *BV* deperdit *Ed*
 14 B *tr. V ante per* / tanto *Ed* tantum
 14 *BV*
 16 continue *VEd om. B*
 17 Praeterea *BV* Probatur sic *Ed* /
 18 difformiter *VEd* distan (!) *B*
 18 presentaretur *BV* representaretur *Ed*
 19 in *Ed om. BV*
 20 suum *BV* etiam suum *Ed* secundum
 21 *BV* applicetur secundum *Ed*
 22 intensiori *BV* remissiori *Ed*
 22-23 suum . . . latitudine *BV* introducere-
 24 tur de latitudine in extremo intensiori
 26 In quorum primo *Ed* primo *BV*
 27 premitte sunt *Ed* premitte *BV* /
 in¹, 2 *om. BV* / conclusiones *BEd*
 questiones *V*
 28 in *om. BV*
 29 illa *B om. Ed* illius *V* / tantam *VEd*
 totam *B*
 31 eiusdem *BEd om. V* / post latitudinis
 add. *Ed* ut patuit in probata ques-
 tione
 31-32 Nota secundo *V tr. Ed* Nota
 primo *B*
 32 idem *BV om. Ed* / est sermo *VEd*
tr. B
 33 et *BV* vel *Ed*
 34 Et *V om. B Ed* / isto *BV* illo *Ed*

secundo notabili nota tertio quod alius est sermo de latitudine motus quantum ad eius esse permansivum et quantum ad eius esse successivum. Nota quarto quod latitudo uniformiter difformis, quo ad eius esse successivum, potest dupliciter acquiri. Potest enim acquiri uniformiter, et etiam potest acquiri difformiter. Uniformiter ad istum sensum: quod si in hora debeat A latitudo uniformiter difformis acquiri, ita quod medietas acquiratur in medietate hore et alia in alia. Et secundum hoc erunt alie et alie conclusiones, et hoc de primo.

Quantum ad secundum, pono aliquas suppositiones. Prima: latitudines formarum per figuras geometricas presentantur. Hanc suppositionem auctor premitit. Secunda suppositio: cuiuslibet trianguli linea duo latera secans per equalia est subdupla ad tertium latus. Patet per quartam sexti Euclidis. Quoniam sit triangulus ABC (vide Fig. 6.12) et linea secans per equalia duo latera, DE , dico quod linea DE est subdupla ad lineam AB . Unde sicut BC ad BE et AC ad AD , sic AB ad ED . Sed BC est duplum ad BE . Ergo AB est duplum ad DE et per consequens linea ED est subdupla ad lineam AB .

Tertia suppositio: quodlibet paralegramon (!) ex linea medii gradus et linea extensionis constitutum est equale triangulo per quem nobis presentatur latitudo uniformiter difformis, cuius linea medii gradus est latus minus dati paralegrami. Hec patet: dato triangulo ABC et linea huius medii gradus DE , tunc fiat paralegramon constitutum ex predictis lineis et sit $BGCF$. Patet suppositio cui nota est geometria. Quarta suppositio: triangulus ABC datus est quadruplus ad triangulum eius partialem. Et ut hoc pateat: sit, gratia exempli,

35 nota BE noto V

36 et BV et alius Ed

38 Potest...acquiri BV dupliciter acquiritur Ed

39 et etiam Ed et B etiam V

40 difformis BV difformiter Ed

41 ita quod Ed que V quod B / hore Ed om. BV

42 post primo add. Ed articulo

43 Prima BV Prima sit hec Ed

44 presentantur BV representantur Ed

45 premitit V premitit in littera Ed premitit B

46 secans $tr.$ V ante est

48 DE BV sit DE Ed

49 sicut B sicut se habet Ed sic V / et

BV ita se habet Ed

50 ED BV DE Ed

50-51 Sed... DE V Ed om. B

51 ED BV DE sive Ed

52 suppositio BV suppositio est Ed / paralegramon BV paralegramum Ed

53 est BE om. V

54 presentatur BV representatur Ed

56 huius BV om. Ed

57 et sit BV om. Ed / $BGCF$ correxi ex

$BGCF$ in MSS ; $BGEG$ Ed / cui nota est geometria BV notata in geometria Ed

59 Et V om. BE / pateat V Ed patefiat

B / gratia exempli Ed om. BV

60 triangulus ABC et linea ED que dividat per equalia duo latera trianguli que latera sint AC et BC . Dico tunc quod triangulus ABC est quadruplus ad triangulum eius partialem qui est DEC . Quod patet resolvendo quadrilaterum $ABDE$ in tres triangulos isto modo: primo ducendo a puncto D usque ad lineam AB eque distantem linee BE et sit hec linea DG . Tunc triangulus AGD equalis est triangulo DEC . Deinde ducatur a puncto D ad punctum B linea BD et habes alios duos triangulos quorum quilibet cuilibet est equalis. Modo patet quod totus triangulus ABC resolutus est in quatuor triangulos equales, et per consequens totus triangulus est quadruplus ad quemlibet illorum et per consequens quadruplus ad triangulum DEC , et hoc fuit declarandum de secundo.

Quantum ad tertium pono conclusiones. Prima: non omnis latitudo uniformiter difformis quantum ad eius esse successivum correspondet suo gradui medio. Probat per primam rationem. Unde sit A unum alterabile quod in hora acquirat sibi latitudinem caliditatis uniformiter difformem, non tamen acquirat eam uniformiter difformiter, sed bene difformiter. Sic scilicet quod in prima quarta hore huius alteretur a non gradu caliditatis usque ad 4 et in reliquis tribus quartis a 4 usque ad 8; tunc, si latitudo acquisita in prima quarta huius hore correspondeat suo gradui medio, A erit alteratum ut duo et si latitudo acquisita in reliquis tribus quartis correspondeat etc., tunc A erit alteratum ut 6 et per consequens in tota hora erit alteratum ut 8. Modo clarum est, quod si A fuisset alteratum ad caliditatem per hanc horam uniformiter ut 8, ergo maiorem latitudinem caliditatis acquisivisset A quam modo sit alteratum. Quare etc. et sic patet quod non omnis latitudo uni-

60 que Ed om. BV / dividat per equalia BE per equalia dividat V

61 latera BV om. Ed

62 est quadruplus BE $tr.$ V / Quod BE om. V

63 $ABDE$ BV $ABDC$ Ed / triangulos BV angulos Ed / isto BV illo Ed

64 primo ducendo $tr.$ B / usque ad Ed

versus BV / AB V Ed om. B

65 et... DG BV om. Ed / equalis est BV

erit equalis Ed

66 habes V sic habes B habebis Ed

67 cuilibet BE cuiuslibet V

68 ABC BV om. Ed / resolutus V Ed

resolutum B

70 post consequens add. Ed est

72 post prima add. Ed sit hec

74 Probat per BV Patet propter Ed /

Unde BV nam Ed

77 hore huius BE $tr.$ V

78 usque V Ed ubique (?) B / a V Ed

ad B

79 huius hore V om. BE

80 suo gradui medio BV etc. tunc Ed

81 reliquis BV aliis Ed / tribus V Ed

duabus sive tribus B / correspondeat

V Ed correspondet B

82 et... 8 $tr.$ Ed post duo in linea 80

84 ergo BE om. V / A BV om. Ed

85 non V Ed om. B

formiter difformis quo ad eius esse successivum correpondet suo gradui medio. Secunda conclusio: omnis latitudo uniformiter difformis uniformiter difformiter acquisita tam quo ad eius esse successivum quam quo ad eius esse permanens correspondet suo gradui medio.

90 Patet per tertiam suppositionem cum auxilio prime. Tertia conclusio: cuiuslibet latitudinis uniformiter difformis incipientis a non gradu vel terminate ad non gradum, gradus medius est precise subduplus ad gradum summum. Patet per secundam suppositionem. Unde dicebatur quod linea DE est precise subdupla ad lineam AB . Et constat

95 quod ille due linee sunt duarum intensionum, quarum una est linea intensionis medii gradus alia est linea intensionis intensioris gradus. Quarta conclusio: nullius latitudinis uniformiter difformis incipientis a certo gradu et terminate ad certum gradum gradus medius est precise subduplus ad gradum summum. Patet per secundam suppositionem: linea AB est precise dupla ad lineam DE , ergo est minor quam dupla ad quamlibet maiorem; sed quolibet linea cadens inter AB et ED est maior quam sit linea DE , ergo conclusio vera. Tenet consequentia, quia cuiuslibet latitudinis incipientis a certo gradu et terminate ad certum gradum, gradus medius cadit ultra lineam medii

100 gradus latitudinis terminate ad non gradum et sic patet conclusio

105

86 correspondet BV correspondeat Ed

92 gradus BEd om. V

93-94 dicebatur VEd patet B

94 precise Ed om. BV

96 intensioris gradus BV intensiorum graduum Ed

99 subduplus BEd duplus V / gradum

BV om. Ed

102 AB BEd A et B V / ED BV CD Ed

103 et BEd om. V

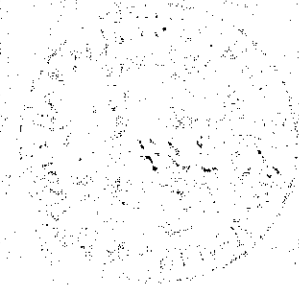
104 cadit BV terminatur Ed

105 terminate BV terminante Ed

NICOLE ORESME

of the University of Paris

Tractatus de configurationibus



NICOLE ORESME

*and the Medieval Geometry of
Qualities and Motions*

A TREATISE

ON THE UNIFORMITY AND DIFFORMITY OF

INTENSITIES KNOWN AS

Tractatus de configurationibus

qualitatum et motuum

EDITED WITH

AN INTRODUCTION, ENGLISH TRANSLATION,

AND COMMENTARY BY

MARSHALL CLAGETT

Uw 563/4 *

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[Incipit prohemium]

[I.o] Cum ymaginationem meam de uniformitate et difformitate intensi-
onum ordinare cepissem, occurrerunt michi quedam alia que huic pro-
posito interieci ut iste tractatus non solum exercitationi prodesset sed etiam
discipline. In quo ea que aliqui alii videntur circa hoc confuse sentire et ob-
scure eloqui ac inconvenienter aptare studui dearticulatum et clare tradere et
quibusdam aliis materiis utiliter applicare. Huius autem tractatus [de figura-
tione potentiarum et mensura difformitatum] tres sunt partes principales.
Prima pars est de figuratione et potentia uniformitatis et difformitatis quali-
tatum permanentium, secunda de figuratione et potentia successivorum, ter-
tia de acquisitione et mensura qualitatum et velocitatum.

Capitula prime partis: Primum de continuitate intensi-
onis. Secundum de latitudine qualitatum. 3^m de longitudine qualitatum. 4^m de quantitate

For title. Title is omitted by VBANCMJSD

Et incipit tractatus de configurationibus
qualitatum et motuum secundum doctorem
et magistrum Nych. Orem. L

Incipit tractatus magistri nicolay oresme de
figuratione qualitatum intyulatus quia
de figuratione et potencia uniformitatis
et difformitatis qualitatum permanentium
etiam successivarum nec non de adquisicio-
ne et mensura velocitatum in tribus parti-
bus huius tractatus pertractat ut patent F

Incipit tractatus de intensione qualitatum
editus per magistrum nicolaum Orijme
qui vocatur etiam tractatus gloriosus G
Incipit tractatus venerabilis Magistri Nycholai
Oresme de configurationibus q(qualita-
tum?) E

De latitudinibus formarum ab Oresme mg.
m. recentiori P, et postea in m. regulari: "Assit
ad inceptum sancta maria mēn"

I.o: Omnes MSS. usque ad lineam 12; (excepto D
qui om. totum prohemium); postea MSS AELF-
MPCNBS

1 [Incipit prohemium] V om. omnes alii MSS.
sed cf. B, f. 171v ubi in tabula capitulorum
legimus Prohemium

2 meam BVFMSGJ om. C veterum AELP
veterum vel meam N / difformitate et uni-
formitate AEP

2-3 intensi-
onum om. B

3 ordinare cepissem tr. S / ordinare: ordi-
nasse G / cepissem: incepissem N / oc-
curunt A / michi om. J. et tr. B post alia /
que om. E / huic om. FM

4 interieci lac. L, om. A intento necessaria E
intento sunt consona P interieci J

4-11 ut....velocitatum: et cetera ut patebit
in tractatu J

4 solum: solummodo C / exercitationi: igno-
rancia exercitationum N cogitationi L

5 quo: que M / ea: illa N / aliqui: qui M ali-
bi G / videntur circa hoc: de hoc videntur
V / circa hoc tr. E post sentire / confuse
sentire om. N confuse L sentire C et confuse
sentire N et tr. N post eloqui / obscure: hoc
obscure A

6 convenienter C / aptari A / dearticulatum...

[Proemium]

I.o. When I began to set in order my¹ conception of the uniformity and dif-
formity of intensities, certain other things occurred to me to add to the topic so
that the treatise might be useful not only as an exercise but also as a discipline.² In
this tract I have attempted to treat clearly and distinctly those matters which some
other people seem to perceive in a confused way, to express obscurely, and to apply
in an unsuitable fashion, and I have attempted to apply them usefully to certain
other matters. This treatise [on the figuration of powers and the measure of dif-
formities]³ has three principal parts: (1) On the figuration and power of the uni-
formity and difformity of permanent qualities. (2) On the figuration and power of
successive things. (3) On the acquisition and measure of qualities and velocities.

Chapters of Part I: 1. On the continuity of intensity. 2. On the latitude of quali-
ties. 3. On the longitude of qualities. 4. On the quantity of qualities. 5. On the figu-

Proemium

¹ Note that I have rejected the reading, "the
view of the ancients," in the poorer tradition
of MSS AELP (see the variant reading for
line 2). See the Commentary, Proemium, line
2, for the significance of *ymaginationem meam*.

² See the Commentary, Proemium, lines 4

and 5.

³ The bracketed title appears to have been a
later addition, as it exists only in MSS AELP.
See Introduction III.D, Passage No. 5, and the
later remarks indicating my belief that it was a
scribal addition.

clare: clare dearticulatumque G / tradere:
ostendere F, (?) M

7 aliis om. FM / utiliter om. B / Huius autem
BVFMSG om. AELPC Huius N

7-8 [de...difformitatum] AELP om. BVF-
MSGN (sed cf. B in tabula capitulorum, f.
173r); var. lect., Cap. III.xiii, lin. 31-32 de
figuratione potentie et de qualitate per
predicta causa quorundam et mensura dif-
formitatum C / figuratione: configuratione
E

8 tres...principales BVFMNSG om. AEP-
C habet tres partes L

9 pars est BVNG pars AELPC est FMS

9-10 qualitatum permanentium BVFMN-
SG om. AELPC

10 secunda BVMSG secunda est V secunda
pars AELNPC / successivorum AEL-
PCSG successivarum BVFMN / tertia:
tertia pars AELPNC

11 qualitatis et velocitatis AELPNC

12-76 Capitula....difformis AELFMPCNS
et B in fine, om. VJG et B in textu hic

12 Capitula....Primum (1 EC) EFMCSB
(sed post partis add. B Prohemium) Capitu-
lum primum (om. A) prime partis APLN /
Primum: Primum capitulum BS / Secun-
dum om. S 2 BEC

13 qualitatum^{1,2} BFMS et cf. I.ii.1 qualitatis
AELPNC / 3^m om. S 3 BEC (hic et pro
omnibus numeris; vide introductionem, p. 145)

- 70 qualitatum et velocitatum difformium. 8^m de mensura et intensione in infinitum quarundam difformitatum. 9^m de quodam [alio] exemplo. 10^m quoddam aliud exemplum de difformitate composita ex partibus uniformibus et uniformiter difformibus. 11^m de mensura et extensione in infinitum qualitatis finite seu velocitatis. 12^m de infinita extensione secundum quid et mensura qualitatis finite et uniformis. 13^m de infinita extensione simpliciter qualitatis finite atque difformis.

[Capitula prime partis]

[I.i] Capitulum primum de continuitate intensionis

- Omnis res mensurabilis exceptis numeris ymaginatur ad modum quantitatis continue. Ideo oportet pro eius mensuratione ymaginari puncta, lineas et superficies, aut istorum proprietates, in quibus, ut vult Philosophus, mensura seu proportio per prius reperitur. In aliis autem cognoscitur in similitudine dum per intellectum referuntur ad ista. Etsi nichil sunt puncta indivisibilia aut lineae, tamen oportet ea mathematice fingere pro rerum mensuris et earum proportionibus cognoscendis. Omnis igitur intensio successive acquisibilis ymaginanda est per lineam rectam perpendiculariter

70 qualitatum *om. L* / qualitatum et velocitatum: velocitatum qualitatis *C* / difformium: difformiter difformium *E* / et *om. S* / intensione *corr. ex* extensione (*vid. cap. III.8*) / in *om. FM*

71 difformitatum: difformitatum difformiter difformibus (*I*) *A* qualitatum *E* / quodam: quorundam *F* / [alio] *supplevi, cf. III.11.1*

71-72 quoddam...de: de quodam alio exemplo et *N*

72-73 de (et *N*)...difformibus *BFMNS om. P* difformitatis compositae ex partibus uniformibus *A* de difformitate *ELC*

73 et: seu *E*

74 seu: vel *N* sive *AE* / secundum: propter *EL* / quid: quam *A*

75 uniformis: uniformitatis *E* / infinita extensione simpliciter *BFM* et *cf. III.11.1* extensione simpliciter infinita *AELNPSC* / qualitatis *om. M*

76 atque: et *N* / *post* difformis *add. B* Expliciunt capitula tertie partis / *post* difformis *add. E* Explicit tabula capitulorum deo gratias / *post* difformis *add. FM* et sic est (erit *M*) finis intencionum omnium capitulorum huius libri divisi in tres partes et cetera / *post* difformis *add. L* Tunc sequitur prima pars

I.i: AEBVDLNFMPSCG[J]

1 [Capitula prime partis] *B, om. alii MSS* prima pars tractatus [*J*]

2 Capitulum...intensionis *BVEG om. ADLNMS[J]* de continuitate intensionis capitulum primum *F* Primum capitulum prime partis *P* Prima propositio *mg. C* 1 *mg. S*

3 Omnes res mensurabiles *NC* / ymaginantur *C*

4 mensurationem *S* mensura [*J*]

qualities and velocities. 8. On the measure and intension to infinity of certain difformities. 9. Another example of the same. 10. A certain other example of a difformity composed of uniform and uniformly difform parts. 11. On the measure and extension to infinity of a finite quality or velocity. 12. On the qualified infinite extension and measure of a finite and uniform quality. 13. On the absolute infinite extension of a finite and difform quality.

Chapters of Part I

I.i On the continuity of intensity

Every measurable thing except numbers is imagined in the manner of continuous quantity. Therefore, for the mensuration of such a thing, it is necessary that points, lines, and surfaces, or their properties, be imagined. For in them (i.e. the geometrical entities), as the Philosopher has it,¹ measure or ratio is initially found, while in other things it is recognized by similarity as they are being referred by the intellect to them (i.e., to geometrical entities). Although indivisible points, or lines, are nonexistent, still it is necessary to feign them mathematically for the measures of things and for the understanding of their ratios.² Therefore, every intensity which can be acquired successively ought to be imagined by a straight line perpendicular-

I.i

¹ See the Commentary, I.i, line 5.

² *Ibid.*, lines 7-9.

4-5 lineas et superficies: superficies et lineas *AP*

4 lineas: linea *N*

5 et: aut *G* / ut...Philosophus: secundum philosophum [*J*] / Philosophus *BVLF-MSCGD* Aristoteles *AENP*

6 seu: et *M* / per prius: proprius *GS(?)* prius *E* / autem: aut *A*

7 dum *BVFMSCG* qua *ENL* que *P* qui *A*

/ per intellectum *tr. N* *post* referuntur / ad ista *tr. N* *post* qua quod stat in loco dum / ista: illa *G* / nichil sunt: non sint *G*

8 mathematice *om. AP* / fingere: sumere *E*

9 proportionibus: proprietatibus et proportionibus *M* / igitur: ergo *G* aut *C*

10 ymaginanda est *tr. FM* *post* intensibilis in linea *11*

11 aliquod punctum *BVFMSCG[J]* aliquot

- 70 qualitatum et velocitatum difformium. 8^m de mensura et intensione in infinitum quarundam difformitatum. 9^m de quodam [alio] exemplo. 10^m quoddam aliud exemplum de difformitate composita ex partibus uniformibus et uniformiter difformibus. 11^m de mensura et extensione in infinitum qualitatis finite seu velocitatis. 12^m de infinita extensione secundum quid et mensura qualitatis finite et uniformis. 13^m de infinita extensione simpliciter qualitatis finite atque difformis.

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70 qualitatum *om. L* / qualitatum et velocitatum: velocitatum qualitatis *C* / difformium: difformiter difformium *E* / et *om. S* / intensione *corr. ex* extensione (*vid. cap. III.8*) / in *om. FM*

71 difformitatum: difformitatum difformiter difformibus (*I*) *A* qualitatum *E* / quodam: quorundam *F* / [alio] *supplevi, cf. III.19.1*

71-72 quoddam...de: de quodam alio exemplo et *N*

72-73 de (et *N*)...difformibus *BFMNS om. P* difformitatis compositae ex partibus uniformibus *A* de difformitate *ELC*

73 et: seu *E*

74 seu: vel *N* sive *AE* / secundum: propter *EL* / quid: quam *A*

75 uniformis: uniformitatis *E* / infinita extensione simpliciter *BFM* et *cf. III.13.1* extensione simpliciter infinita *AELNPSC* / qualitatis *om. M*

76 atque: et *N* / *post* difformis *add. B* Expliciunt capitula tertie partis / *post* difformis *add. E* Explicit tabula capitulorum deo gratias / *post* difformis *add. FM* et sic est (erit *M*) finis intencionum omnium capitulorum huius libri divisi in tres partes et cetera / *post* difformis *add. L* Tunc sequitur prima pars

Ii: AEBVDLNFMPSCG[I]

1 [Capitula prime partis] *B, om. alii MSS* prima pars tractatus [*J*]

2 Capitulum...intensionis *BVEG om. ADLNMS[I]* de continuitate intensionis capitulum primum *F* Primum capitulum prime partis *P* Prima propositio *mg. C* 1 *mg. S*

3 Omnes res mensurabiles *NC* / ymaginatur *C*

4 mensurationem *S* mensura [*J*]

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5 et: aut *G* / ut...Philosophus: secundum philosophum [*J*] / Philosophus *BVLFMSCGD* Aristoteles *AENP*

6 seu: et *M* / per prius: proprius *GS(?)* prius *E* / autem: aut *A*

7 dum *BVFMSCG* qua *ENL* que *P* qui *A*

/ per intellectum *tr. N* *post* referuntur / ad ista *tr. N* *post* qua quod stat in loco dum / ista: illa *G* / nichil sunt: non sint *G*

8 mathematice *om. AP* / fingere: sumere *E*

9 proportionibus: proprietatibus et proportionibus *M* / igitur: ergo *G* aut *C*

10 ymaginanda est *tr. FM* *post* intensibilis in linea *II*

11 aliquod punctum *BVFMSCG[I]* aliquot

erectam super aliquod punctum spatii vel subiecti illius rei intensibilis, ut, verbi gratia, qualitatis. Nam quaecunque proportio reperitur inter intensio-
nem et intensionem de intensionibus que sunt eiusdem rationis similis pro-
portio invenitur inter lineam et lineam et e contra; quemadmodum enim una
linea alteri linee est commensurabilis et alteri incommensurabilis, ita est con-
formiter de intensionibus quod quedam sunt commensurabiles adinvicem et
quedam incommensurabiles quomodolibet propter continuitatem earundem.
Ergo mensura intensionum potest ymaginari congrue sicut linearum mensura,
cum etiam intensio possit eodem modo sicut linea in infinitum diminui
et quantum est ex se in infinitum augeri.

Rursum intensio secundum quam aliquid dicitur magis tale, ut magis album aut magis velox; ipsa quidem, secundum quod intensio vel intensio puncti est tantum uno modo divisibilis et in infinitum ad modum continui, igitur non potest convenientius ymaginari quam per illam speciem continui que est primo divisibilis et uno modo tantum, scilicet per lineam. Et quoniam linearum quantitas sive proportio notior est et facilius a nobis concipitur, ymmo linea est in prima specie continuorum, ideo per lineas ymaginanda est intensio talis—maxime vero et convenientissime per illas que subiecto applicate super ipsum perpendiculariter eriguntur, quarum consideratio ad cuiuslibet intensionis notitiam naturaliter iuvat et ducit, prout ex 4^o capitulo sequenti plenius apparebit. Ideoque intensiones equales per equales lineas designantur et dupla intensio per duplam lineam et sic semper proportionaliter procedendo. Et istud est universaliter intelligendum de omni intensione ad ymaginationem divisibili, sive sit intensio qualitatis active sive non active, sensibilis sive insensibilis subiecti aut obiecti aut medii, ut de luce

punctum *N* aliquod punctum aut (vel *E*) aliquot puncta *AEP* aliquot puncta *LD* / spatii: intensibilis spatii *AP* / subiecti: spatii subiecti *B* / illius...intensibilis *om.* *AP* / rei *om.* *F*

11–12 ut...qualitatis *VSG* ut gratia qualitatis *C* verbi gratia ut qualitas *FM* ut verbi gratia *BLN* verbi gratia *AEP*

12 Nam. *om.* *LN* *lac.* *V* / quaecunque: qualiscunque *AELNP* / reperitur: invenitur *E*

13 et intensionem *om.* *FM* / que: qualitatis (?) que *B* quod quedam *F*

14 invenitur: reperitur *VAE* / et lineam *om.* *FM* / e contra: e converso *FM* / enim *om.* *AP* nisi *C*

14–15 una linea *tr.* *S*

15 linee *om.* *AL* / est *tr.* *VM* post linea / et... incommensurabilis *om.* *C* / et *om.* *PCFM*

17 quedam: quidem *S* / quolibet *M* / earundem *ABENP* earum *VS* eorum *FMCG*

eorundem *L*

18 Ergo: igitur *AVES* / ymaginari congrue *tr.* *N*

19 possit: posset *M* / possit...modo: eodem modo possit *C* / diminui: dividi et diminui *FM* / diminui *tr.* *C* ante sicut

20 quantum: quantitas *FM* / ex: de *AP* / in infinitum *om.* *G*

21 Rursum: Et rursum *AELNP* / post intensio *sup.* *scr.* *m.* *rec.* *C* est / aliquid: aliquod *AP* / dicitur *B* / magis tale: magis clare *A* et add. *A* ante dicitur / tale: tale aut minus tale *E* / ut magis: et minus *A* ut minus *P*

22 post album add. *F* aut niger / aut: vel *V* / magis: minus *E* / intensio¹: extensio *A* est intensio *S*

22–23 vel intensio puncti *ABVCS* *om.* *FM* ut intensio puncti *G* vel extensio puncti *LNP* ut extensio puncti *E*

23 tantum: cum *L* / et: etiam *G*

ly erected on some point of the space or subject of the intensible thing, e.g., a quality. For whatever ratio is found to exist between intensity and intensity, in relating intensities of the same kind, a similar ratio is found to exist between line and line, and vice versa. For just as one line is commensurable to another line and incommensurable to still another, so similarly in regard to intensities certain ones are mutually commensurable and others incommensurable in any way because of their [property of] continuity. Therefore, the measure of intensities can be fittingly imagined as the measure of lines, since an intensity could be imagined as being infinitely decreased or infinitely increased in the same way as a line.

Again, intensity is that according to which something is said to be "more such and such," as "more white" or "more swift." Since intensity, or rather the intensity of a point, is infinitely divisible in the manner of a continuum in only one way, therefore there is no more fitting way for it to be imagined than by that species of a continuum which is initially divisible and only in one way, namely by a line. And since the quantity or ratio of lines is better known and is more readily conceived by us—nay the line is in the first species of continua, therefore such intensity ought to be imagined by lines and most fittingly by those lines which are erected perpendicularly to the subject. The consideration of these lines naturally helps and leads to the knowledge of any intensity, as will be more fully apparent in chapter four below. Therefore, equal intensities are designated by equal lines, a double intensity by a double line, and always in the same way if one proceeds proportionally. And this is to be understood universally in regard to every intensity that is divisible in the imagination, whether it be an active or non-active quality, a sensible or non-sensible subject, object, or medium. For example, it is to be understood in regard to the light of the body of the sun, to the illumination of a medium, or to a species in the medium, to a diffused influence or power, and similarly to others, with the

24 igitur *VLNS* ergo *BFGM* et ideo *AEP* / igitur...continui *om.* *C* / convenientius ymaginari *tr.* *V*

25 que: qui *A* / est primo *BVFMS* *tr.* *AELNP* *CGD*

26 sive: seu *A* / est *om.* *N*

26–27 concipitur: percipitur *M* corrumpitur (1) *C* sed add. *m.* *rec.* *mg.* vel concipitur

27 ymmo: ymo *FNA* ideo *C* / linea est: primo est linea *E* / lineas: lineam *S*

27–28 ymaginanda est *tr.* *N*

28 vero: enim *VS* (?) / per illas *om.* *G* per illos *C*

29 super ipsum *om.* *G* / ipsum *om.* *C* ipsam *L* / eriguntur quarum *tr.* *L* / quarum: quorum *V* quare *A* et *S* / ad: a *P*

30 cuiuslibet: cuiuscunque *BVG* cuiuscuiuslibet *C* / naturaliter iuvat *tr.* *G* / ex: in *AELPC* / 4^o capitulo *tr.* *N* primo capitulo *C*

31 apparebit: patebit *A*, et *mg.* add. *C* conclusio

31–32 per...intensio *om.* *AP*

32–33 proportionaliter: proportionabiliter *CG*

33 istud: illud *N* / est universaliter *tr.* *V*

34 divisibilis *A* / sit: sit etiam *V* / intensio *om.* *FM*

35 ante sensibilis add. *AELNP* sive / insensibilis: non sensibilis *C* / aut¹: sive *APM* vel *BF* / aut²: vel *BVSG* / de *om.* *CSG*

35–36 luce corporis: corporis *G* corpore *VC*

corporis solis et de lumine medii, vel de specie in medio, vel influentia aut virtute diffusa, et sic de aliis, excepta forsitan intensione curvatis, de qua dicetur ad partem in capitulis 20^o et 21^o huius partis.

Huiusmodi vero linea intensiois de qua nunc dictum est non extenditur extra punctum vel extra subiectum secundum rem sed solum secundum ymaginationem, et ad quamvis partem nisi quod convenientius ymaginatur in sursum perpendiculariter stare super subiectum qualitate informatum.

[Lii] Capitulum secundum de latitudine qualitatum

Omnis intensio per predictam lineam designata proprie vocari deberet longitudo illius qualitatis, primo quidem quia in alteratione continua [essentialiter] non exigitur successio secundum extensionem sive secundum partes subiecti [quia potest totum simul incipere alterari] sed ibi requiritur successio secundum intensioem. Ergo sicut in motu locali illa dimensio dicitur longitudo spatii seu vie secundum quam exigitur successio, ita conformiter huiusmodi intensio secundum quam requiritur successio deberet dici longitudo ipsius qualitatis. Item sicut velocitas in motu locali secundum longitudinem spatii mensuratur, ita in alteratione velocitas attenditur penes intensioem. Ergo talis intensio deberet dici longitudo. Item nulla qualitas alteratione acquisibilis potest ymaginari sine intensioe seu divisibilitate secundum intensioem, sed bene potest ymaginari sine extensione; ymmo qualitas subiecti indivisibilis, ut anime vel angeli, non habet extensionem. Cum igitur ymaginetur mathematica longitudo sine latitudine et non e converso, et intensio sit referenda ad aliquam dimensionem, ut patet ex

prius (?) corporis S

36 et: aut E / vel¹: et VE / vel²: vel de E / aut om. FM vel B

37 virtute diffusa: divina virtute infusa FM / forsitan: forte N / de curvatis scr. m. rec. C in mg. vel caritatis

38 ad partem: post N partialiter (?) E / in om. S / capitulo V / 20^o et: 2^o N 7^o C / huius: istius V / huius partis om. G

39 Huiusmodi: huiusmodi cot (?) C / vero: vera C / extendit C

40 extra¹: ultra M / subiectum: san¹ (?) C / solum om. AP

41 in om. AEP

42 sursum: situ sive S / informatum VEM-NSG formatum (?) B, AFLPCD

Iji: BVAEDLNFMPCSG

1 Capitulum secundum BVELPSCD om. ANM, tr. FG post qualitatum / de... qualitatum BVE(?) FG om. ANPMS de latitudine qualitatis LD de velocitate qualitatis C

2 Omnis BVSCGM Omnis igitur AELPD Omnis ergo FN / designata: ymaginata FM distincta A / vocari deberet BLND vocari debet VFMSG debet vocari EP deberet vocari A

3 longitudo: eius longitudo C / illius om. F / quidem: igitur quidem L igitur N / quia om. N

3-4 [essentialiter] AELNPD om. BVFMSG

4 exigitur: requiritur E / secundum² om. N

possible exception of curvature, concerning which we shall speak in a limited way in chapters twenty and twenty-one of this part [of our work].

Of course, the line of intensity of which we have just spoken is not actually extended outside of the point or subject but is only so extended in the imagination, and it could be extended in any direction whatever except that it is more fitting to imagine it standing up perpendicularly on the subject informed with the quality.³

I.ii On the latitude of qualities

Every intensity designated by the aforesaid line ought properly to be called the longitude of the quality.¹ This is primarily because in continuous alteration succession according to the extension or parts of the subject is not [essentially] demanded, [for the whole subject can begin to be altered simultaneously], but succession according to intensity is required there. Therefore, just as in local motion that dimension according to which succession is demanded is called length of space or path, so similarly intensity of this sort according to which succession is required should be called the longitude of this quality. Also, just as velocity in local motion is measured according to length of space, so velocity in alteration is a function of intensity. Therefore, such intensity should be called longitude. Also, no quality acquirable by alteration can be imagined without intensity or divisibility according to intensity, but it can well be imagined without extension. Nay, a quality of an indivisible subject, such as a soul or an angel, does not have extension. Since, therefore, length is imagined mathematically without breadth but not conversely, and since intensity ought to be referred to some dimension, as is evident in the

³ Ibid., lines 40-41.

I.ii

¹ See the Commentary, I.ii, lines 2-24.

5 [quia...alterari] AELNPD om. BVFMSG / potest totum EP tr. LND possunt (?) totum A / ibi om. AEP

6 Ergo: igitur LND ideo APE / dicitur VFMSG dicitur BC diceretur AELNPD

7 seu: sive P

8 intensio: enim intensio L / debet (?) BC ipsius: illius FNMC et tr. N post qualitatis

/ Item: Et NS / locali om. E

10 spatii tr. V post mensuratur

11 Ergo: igitur VNSG / debet (?) VCE / Item: 3^o N / post qualitas add. P neque / post qualitas add. E in

12 alteratione: alterative G / seu: sex (?) V aut EAP sive S / divisibilitate: difformitate A diversitate LD ex

13 bene potest tr. AE / extensione: intensioe L

14 post anime add. E intensive / habent FS

15 ymaginetur: imaginatur AEP / mathematica tr. AEP post latitudine / longitudo: loquendo L / et om. B

16 et: et cum ELNP et tamen A / referenda: differenda A / ex: in AELNP

17 precedenti capitulo tr. AN / est: esset BSG / non: et non AELNPG

precedenti capitulo, ipsa referenda est ad longitudinem non ad latitudinem, et nomine longitudinis magis proprie appellanda. Unde patet quod qualitas subiecti indivisibilis proprie non habet latitudinem. Sed multi theologi loquuntur improprie de latitudine caritatis, quia si per latitudinem intelligunt intensionem, tunc contingeret invenire latitudinem sine longitudine, et sic eorum transsumptio videretur incongrua. Verumptamen huiusmodi intensionem vocabo latitudinem qualitatis, sicut plenius dicam in capitulo immediate sequenti.

[I.iii] Capitulum 3^m de longitudine qualitatum

Cuiuslibet qualitatis extense sua extensio deberet vocari ipsius latitudo et predicta extensio designatur per lineam in subiecto descriptam super quam linea intensionis qualitatis eiusdem perpendiculariter erigitur. Nam cum omnis talis qualitas habeat intensionem et extensionem que in eius mensura sunt attendende, ideo si eius intensio diceretur longitudo tunc extensio, que esset secunda dimensio, vocaretur latitudo. Et etiam econverso si intensio dicatur latitudo extensio vocabitur longitudo. Sicut igitur corporis vel superficies lineam longitudinis et lineam latitudinis perpendiculariter sibi invicem adjacent, ita etiam extensio qualitatis que deberet dici ipsius latitudo ymaginanda est per lineam perpendiculariter adjacentem linee longitudinis qualitatis eiusdem. Et quemadmodum in permanentibus extensio in subiecto debet dici latitudo qualitatis et intensio longitudo, ita conformiter in successivis, cuiusmodi sunt motus, sonus, et similia, extensio eorum in tempore vo-

18 longitudinis: latitudinis *E* / magis proprie: proprius *BVCSG* / appellanda: vocanda *N*

19 Sed *AELNPD* sicut *BVFMCSG* / multi: quidam *M*

20 intelligunt: intelligant *A* intelligerent *E*

21-22 invenire.... Verumptamen: —rump- tamen *V*

22 earum *G* / videretur *BFMG* videtur *AELNPCD* / videretur incongrua *om. S*

22-24 Verumptamen... sequenti: vocabulo tamen latitudinem sicut ipsi ut magis intelligantur que dicam et propter consuetudinem locandi sic vocabulo *D*

22-23 post intensionem add. *BVSFMC* aliquotiens sed *om. AELNP*; habet *G* aliquando sed *tr. G* ante latitudinem

23 sicut... dicam: prout dicam plenius *AP* sicut dicam plenius *E* / sicut: ut *N* sic *S*

23-24 immediate *om. AENG* isto *P*

I.iii: BV AEDLNFMPCSG

1 Capitulum... qualitatum (qualitatis *LD*) *BVELMD om. AN* 3^m capitulum *PS* De longitudine qualitatum capitulum 3^m *FG* Tertium capitulum de longitudine qualitatis *C* 3 *mg. A*

2-4 Cuiuslibet.... Nam *om. D*

2 extense sua: intense seu intentio *F* intense sua intensio *M* / debet *C* / ipsius *BVFMCSG om. AP* sua *ELNC et tr. E* post latitudo / ipsius latitudo *tr. G*

3 designatur *BVFMCSG* designari potest *ALNP* designari *C* potest designari *E*

4 qualitatis *om. E* / eiusdem *om. V* / erigatur *S* / Nam: modo *P* quia *N* / cum: tamen *E*

5 omnis *om. A* / talis *om. D*

preceding chapter, it (intensity) ought to be referred to length not to breadth, and it more properly ought to be called by the name of longitude rather than by that of latitude. Thus it is clear that the quality of an indivisible subject does not properly have latitude. But many theologians speak improperly of the "latitude of charity"²—for if by "latitude" they understand intensity then breadth would be found without length, and so their transference of the meaning would seem unfitting. Be that as it may, I shall call³ intensity of this kind the "latitude" of quality, as I shall declare more fully in the chapter that immediately follows.

I.iii On the longitude of qualities

The extension of any extended quality should be called its latitude. The aforesaid extension is designated by a line drawn in the subject, a line on which the line of the intensity of the same quality is erected perpendicularly. For since every quality of this sort has intensity and extension on which its measure is dependent, therefore if its intensity were called longitude, then its extension—which would be its second dimension—would be called [its] latitude. And also conversely, if intensity is called latitude, extension will be called longitude. Therefore, just as the line of length and that of breadth of a surface or body are perpendicularly adjacent to one another, so also the extension of a quality, which should be called its latitude, ought to be imagined by a line perpendicularly adjacent to the line of longitude of the same quality. And just as in permanent qualities extension in the subject ought to be called the latitude of the quality and intensity longitude, so similarly in successive things—of which sort are motion, sound, and the like—their extension in time

² *Ibid.*, lines 19–20.

³ Or, "sometimes call." See the variant reading. I have chosen the reading of the poorer

tradition primarily because, in fact, Oresme always calls intensity by the name *latitudo*.

6 sunt attendende: sit attendenda *AELPD* / ideo: igitur *V*

6-7 diceretur... intensio *om. V*

6 longitudo: extensio *AP*

7 vocabitur *S* / etiam *om. FM*

7-8 etiam... longitudo: econtra *D* eius

8 extensio: tunc extensio *N* eius et extensio *V* / vocatur *G* / igitur *om. V* ergo *N*

8-9 corporis vel superficies *om. D*

8 vel *om. AP*

9 et: vel *LM* / linea *om. D* / sibi: se *AEP*

9-12 sibi... eiusdem: se dividant ita et ex-

tensio qualitatis et intensio *D*

10 adjacent: dividunt *AELP* / etiam: est *G* / extensio: intensio *FM* / ipsius: eius *AEPG* / latitudo: longitudo *FMCSG*

11 longitudinis: latitudinis *FM*

12 quemadmodum: sicut *N* / in subiecto *om. AELPCD*

13 debet dici *tr. L* / intensio longitudo: extensio latitudo *V*

14 sonus: et sonus *AELPCD* / similia: consimilia *SG* cetera *N* / earum *E* / in tempore *tr. G* post vocaretur

15 caretur latitudo et intensio longitudo. Verumtamen quia extensio est manifestior et palpabilior, ut ita loquitur, et prior cognitione quo ad nos quam sit intensio, et forsitan quo ad naturam, ideo non obstantibus predictis ipsa extensio secundum communem usum loquendi attribuitur prime dimensioni, scilicet longitudini, et intensio latitudini. Et quoniam differentia
 20 huiusmodi impositionis seu improprietas vocationis nichil facit ad rem sed utroque modo potest idem exprimi, volo sequi modum communem ne propter locutionem inconsuetam illa que dicam minus leviter intelligantur. Extensio igitur qualitatis in nomine dei vocetur eius longitudo et intensio
 25 ipsius vocetur latitudo sive altitudo. Sed qualitercunque sit, patet ex dictis quod quidam moderni non bene vocant latitudinem qualitatis ipsam totam, sicut abusio esset per latitudinem superficiei intelligere totam superficiem vel figuram. Nam quemadmodum aliquae latitudines superficierum sive figurarum inequalium sunt equales ita similiter, sicut postea videbitur, multae latitudines qualitatum inequalium sunt equales aut etiam econverso.

[I.iv] Capitulum 4^m de quantitate qualitatum

Cuiuslibet linearis qualitatis quantitas ymaginanda est per superficiem cuius longitudo seu basis est linea in subiecto quali protracta, ut dicit precedens capitulum, et cuius latitudo seu altitudo designatur per lineam super
 5 basim predictam perpendiculariter erectam secundum quod ponit capitulum

- 15 intensio: intensio vocaretur *N* / Verumtamen *APCE* verum *BVFM* *SG* verum est tamen *LND* / quia: quod *LND* / extensio: intensio *LND*
 16 et² *om. S* / prior: peior *G*
 16-17 quam sit intensio *om. D*
 17 intensio: extensio *N* / et *om. AP* / ideo: et ideo *FM* ergo *D* / ipsa *om. AP*
 18 usum: modum *AEP*
 19 scilicet: et i. (id est?) *V* / longitudo *S*
 19-22 Et...intelligantur *om. D*
 19 Et: Sed *F* / quoniam: quia *AEP* / quoniam differentia *tr. L* / differentia: natura *FM*
 20 huiusmodi: huius *B(?)VFM* / seu: vel *AEP* / proprietas *VFM* / vocationis: locutionis *P*
 21 ne: nec *L*
 22 locutionem: modum loquendi *AP* loquationem *B* / inconsuetum *AP* / illa: ista *N* / intelligitur *N*
 23 igitur: enim *V* ergo *S* / dei: domini *NFM*
 23-24 eius...vocetur: ipsius *A*
 23 eius *om. D*
 24 ipsius: eius *LS* et *tr. S* ante intensio / ipsius *tr. G* ante latitudo / vocetur *ENPFM om. D* vocatur *L* dicatur *BVSCG* / latitudo sive altitudo: altitudo vel latitudo *P* / sive: seu *ED* vel *AP*
 24-29 Sed...econverso *om. D*
 24 sit *om. N* / ex dictis *om. V*
 25 bene vocant *tr. A* quidem bene vocant *S*
 26 esset: est *PLN* / latitudinem superficiei *tr. N* / intelligere...superficiem *om. S*
 27 vel: sive *FM* aut *A* / Nam quemadmodum: non quemadmodum *S* quia sicut *AEP* quia quemadmodum *N* / aliquae: quedam *FM* / aliquae latitudines *tr. V* / sive: aut *A* vel *LP*

should be called latitude and their intensity longitude. Nevertheless, extension as it is thus spoken of is more manifest, more palpable, and prior in our cognition than is intensity.¹ It is perhaps also prior in nature. Therefore, notwithstanding my previous statements, this extension according to the common practice of speech is associated with the first dimension, namely longitude, and intensity with latitude. And since a difference in the application of [a name of] this sort, or an impropriety in naming, actually has no effect and the same thing can be expressed in either way, I wish [accordingly] to follow the common way. I do this so that those things which I say might not be less easily understood because of unaccustomed locution. Therefore, in the name of God let the extension of a quality be called its longitude and intensity its latitude or altitude. But however this might be, it is obvious from the things said that certain moderns do not speak in the best way when they call the whole of the quality its latitude, just as it would be an abuse [of terminology] to understand by the breadth of a surface the whole surface or figure.² For just as the breadths of some unequal surfaces or figures are equal, so, as will be seen later, many latitudes of unequal qualities are equal, or vice versa.

I.iv On the quantity of qualities

The quantity of any linear quality is to be imagined by a surface whose length or base is a line protracted in a subject of this kind, as the preceding chapter says, and whose breadth or altitude is designated by a line erected perpendicularly on the

I.iii

¹ See the Commentary, I.iii, lines 15-17.

² *Ibid.*, lines 24-27.

28 postea videbitur: patebit postea *AEP*
 29 aut: vel *P* / aut...econverso *om. FM* / post econverso *add. E* et cetera

I.iv: *BVAPFLSD*—collated throughout; [*MENCGJ*]—also used, but not completely, and then placed in brackets
 1 Capitulum...qualitatum *LBVD[CE] om. AS[MN]* 4^m capitulum *P* De quantitate qualitatum capitulum 4^m *F[G]* 4 c *mg. S* 4 *mg. A*

2 linearis (linealis *B*)...quantitas *BFS* [*GNM*] qualitatis quantitas linealis (linearis *L[ECD]*) *ALP[CDE]* quantitas linealis qualitatis *V* ymaginanda est *tr. AP[E]*
 3 seu: vel *AP[E]* / pertracta *P[M]*
 3-4 precedens capitulum *tr. AP[E]*
 4 seu *LPDF[EMN]* sive *BV[CG]* sine *S*
 5-6 predictam...qualitatem²: aliquem *A*
 5 predictam: productam *LPD[EN]* / secundum...ponit: sicut dicit *P[E]* ut ponit [*N*] sicut ponit *D*

secundum. Et intelligo per qualitatem linearem qualitatem alicuius lineae in subiecto informato qualitate.

Quod enim quantitas talis qualitatis per huiusmodi superficiem possit ymaginari patet, quoniam contingit dare superficiem illi qualitati equalem in longitudine seu extensione et similem in altitudine eidem qualitati in intensione, ut patebit post. Sed quod per hoc debeamus ymaginari qualitatem ut eius dispositio levius cognoscatur apparet quia eius uniformitas atque difformitas citius, facilius, et clarius perpenduntur quando in figura sensibili aliquod simile describitur quod ab ymaginatione velociter et perfecte capitur et quando in exemplo visibili declaratur. Satis enim difficile videtur quibusdam intelligere que sit qualitas uniformiter difformis. Sed quid facilius quam quod trianguli rectanguli altitudo est uniformiter difformis? Certe hoc apparet ad sensum. Cum igitur intensio huiusmodi qualitatis per altitudinem talis trianguli fuerit figurata et ei assimilata sicut fiet in 8^o capitulo, tunc de facili cognoscetur huiusmodi qualitatis difformitas, dispositio, figuratio, et mensura, et ita de aliis. Nec alio modo possent species seu diversi modi difformitatis agnosci nec aliter assignari, sicut patebit capitulis 14^o et 15^o huius partis. Multum enim iuvat ad cognitionem rerum ymaginatio figurarum, propter quod theologi dicunt illud fuisse figuram alicuius rei ex cuius similitudine in illius rei notitiam poterat deveniri et configurari ei et assimilari. Nam ut dicit Apostolus de Christo quod "reformabit corpus humilitatis nostre, configuratum corpori claritatis sue." Glosa dicit "i.e., assimilabimur corpori illius in claritate."

Nunc autem ad propositum revertendo, sicut punctualis qualitas ymaginatur ut linea et linearis per superficiem, ita qualitas superficiei ymaginatur ut

- | | |
|--|--|
| 6 per om. <i>F[M]</i> / linearem: linealem <i>P</i> / qualitatem ^a om. <i>P</i> | 16 quam: enim <i>A</i> |
| 8 enim quantitas tr. <i>A</i> / talis qualitatis tr. <i>LD[E]</i> talis <i>A</i> / huiusmodi superficiem tr. <i>B</i> | 17 quod: quo <i>AP</i> |
| 9 quoniam: quia <i>AP[EN]</i> | 17-23 Certe....partis: cui comparatur talis qualitas <i>D</i> |
| 10 eiusdem <i>S</i> / quantitati <i>P</i> / in om. <i>L[N]</i> | 17-18 apparet: patet <i>A[GN]</i> |
| 11 patebit post tr. <i>P[N]</i> patebit <i>[G]</i> / debemus bis <i>P</i> | 18 huiusmodi om. <i>F[M]</i> |
| 11-15 Sed....declaratur om. <i>D</i> | 19 trianguli: trianguli rectanguli altitudo est uniformiter difformis <i>L</i> / 8 ^o : 9 ^o <i>AP</i> / capitulo om. <i>S</i> |
| 12 dispositiones <i>ALP[EN]</i> / cognoscantur <i>ALP[EN]</i> / eius uniformitas tr. <i>F[M]</i> / atque: et <i>AP[EN]</i> | 20 cognoscantur <i>FB</i> / qualitatis difformitas tr. <i>A</i> qualitatis difformitas et difformis <i>L</i> |
| 13 citius: citius et <i>L[N]</i> / et om. <i>B</i> | 21 et...aliis om. <i>APL[EC]</i> et cetera <i>[N]</i> / species: superficies <i>A</i> / seu om. <i>AP</i> / diversi modi: diversimode <i>P</i> |
| 14 aliquod: ad(?) <i>S[C]</i> aliquid <i>[NG]</i> | 22 agnosci: cognosci <i>F[M]</i> / capitulis: in capitulis <i>APL[E]</i> |
| 15 et om. <i>AP</i> / enim om. <i>D</i> | 23 enim: si (?) <i>A</i> |
| 15-16 quibusdam intelligere om. <i>AP</i> | 24 illud <i>BVFS[MG]</i> illam <i>ALPD[ECN]</i> / alicuius rei om. <i>A</i> alicuius <i>S[M]</i> |
| 16 que: quid <i>LPD[EC]</i> quod <i>A</i> | 25 notitiam tr. <i>V</i> post in / devenire <i>F[MN]</i> |
| 16-17 Sed...difformis <i>BVAPDF[MECN]</i> , mg. <i>L</i> , om. <i>S[G]</i> et text. <i>L</i> | |

aforesaid base in the way that the second chapter proposes. And I understand by "linear quality" the quality of some line in the subject informed with a quality.

That the quantity of such a [linear] quality can be imagined by a surface of this sort is obvious, since one can give a surface equal to the quality in length or extension and which would have an altitude similar to the intensity of the quality, as will be clear later. But it is apparent that we ought to imagine a quality in this way in order to recognize its disposition more easily, for its uniformity and its difformity are examined more quickly, more easily, and more clearly when something similar to it is described in a sensible figure. [This is true] because something is quickly and perfectly understood when it is explained by a visible example. Thus it seems quite difficult for certain people to understand the nature of a quality that is uniformly difform. But what is easier to understand than that the altitude of a right triangle is uniformly difform? For this is surely apparent to the senses. Therefore, when the intensity of a quality of this sort is figuratively represented by the altitude of such a triangle and is assimilated to it in the manner done in chapter eight, then one recognizes with ease in such a quality its difformity, disposition, figuration, and measure¹; and similarly for other qualities. Now there is no other way by which the species and diverse modes of difformity could be recognized and otherwise assigned, as will be obvious in chapters fourteen and fifteen of this part. For the imagining of figures is a great help in the understanding of things. Accordingly, theologians say that it was from the similarity of the figure of something that one was able to come into knowledge of that thing and to be conformed and assimilated to it. For as the Apostle says concerning Christ, He "will reform the body of our lowliness, made like to the body of His glory."² A gloss on this passage says: "i.e. we shall be assimilated to His body in glorification."³

Now, reverting to the subject at hand, just as the quality of a point is imagined as a line, and the quality of a line by a surface, so the quality of a surface is imagined

I.iv

¹ See the Commentary, I.iv, lines 8-26.

² *Epist. to the Philippians* 3:20-21.

³ The so-called *Glossa ordinaria*. See the Commentary, I.iv, lines 27-28.

25-28 et¹....claritate *APLS[CEG]* om. *VBF[M]*

25-26 et¹....Apostolus: configurari enim i. assimilari unde apostolus ait *S*

25 configurari: figurari *A*

26-44 Nam....apparebit om. *D*

27-29 configuratum....revertendo: et cetera dico igitur quod *[N]*

27 Glosa *L[C]* glo^a diffinitive *A* glo^a *PS[EG]* / i.e. *S[CG]* cf. *glos. ord.* ibi *ALP[E]*

27-28 assimilabimur corpori *glos. ord.* assimilari corpori *P[E]* assimilatum corpori *S[G]* assimilatio in corpora *A* assimilatum corpus *L[?C]*

28 claritate *S[CG]* cf. *glos. ord.* glorificatione *ALP[E]*

29 Nunc autem: Sed nunc *AP[E]* / punctualis: patet talis *F[M]*

30 linea....ut om. *AP*

corpus cuius quidem corporis ymaginati basis est superficies ipsa informata qualitate prout plenius declarabitur in processu. Cum autem in corpore quali infinite sint superficies [equales] et cuiuslibet earum qualitas ymaginetur ut corpus, non est inconueniens, sed oportet, ymaginari unum corpus
 35 secundum situm ubi aliud potest ymaginari simul, vel etiam quodlibet simul, per penetrationem vel per mathematicam suppositionem (/superpositionem?) seu simul positionem corporum sic fictorum; que tamen penetratio non est in re. Et quamvis qualitas superficialis ymaginetur per corpus, et non contingat esse vel ymaginari quartam dimensionem, tamen qualitas corporalis
 40 ymaginatur habere duplicem corporeitatem: unam veram ad extensionem subiecti secundum omnem dimensionem, aliam vero solum ymaginatam ab intensione ipsius qualitatis infinities replicabilem secundum multitudinem superficierum subiecti, cuius ymaginationis op(p)ortunitas prius tacta est et in sequentibus plenius apparebit.

[I.v] Capitulum 5^m defiguratione qualitatum

Omnis qualitas linearis figuratur ad modum alicuius superficiei super subiectam lineam perpendiculariter erecte. Sit enim *AB* linea informata qualitate [Fig. 1]. Et quoniam per precedens capitulum qualitas ista designatur per superficiem, oportet quod ymaginetur figurata sicut superficies per quam ipsa designatur vel ymaginatur. Cuius quidem superficiei altitudo designat intensionem istius qualitatis. Oportet etiam quod istius superficiei

31 superficies ipsa *tr. A*

32 prout...declarabitur: sicut plenius patebit *A* sicut patebit plenius *P[E]* ut post declaratur [*N*] / in processu *om. [N]* / Cum: nunc *F[M]* / autem: igitur *V*

33 sint: sunt *AF[M]* / [equales] *BVAPSL [EGCN]*, sed delendum est, sicut in *F[M]*?

33-34 ymaginatur *F[MN]*

34 sed oportet *om. L* / sed: sed forte *F[MN]* / post ymaginari *add. S* sit

35 secundum situm: sic fictum *F[N]* situm *S* sic infinitum [*C*] / ubi...simul¹: simul esse ubi aliud *AP[E]* / vel: aut *A* / quodlibet: quomodolibet *F[M]* quolibet *S* quotlibet [*E*] / simul²: sive *A*

36 penetrationes *L[C]* / vel: seu *VF* / per *BSF[CGM]* *om. V* etiam per *AP[EN]* etiam *L* / suppositionem: positionem *V*

37 positionem: possent *V* / sic *om. AL* / fictorum *BVSF[MN]* situatorum *APL[E]* sitorum [*G*] finitorum [*C*] / post fictorum *add. L* secundum infinitatem

38 qualitas *om. L* / ymaginatur *P[N]* / et²: tamen *L* et tamen [*N*]

38-39 contingat *BVFS [MGN]* conting^t *APL* contingit [*EC*]

39 tamen *VPLS[EGC]* cum *BAF[M,?N]*

40 post duplicem *add. S* ipse

41-42 aliam...multitudinem *om. S*

41 vero *om. F[M]* / solum *VAPF[EGC]* solam *BL[MN]* / solum ymaginatam *tr. A*

42 replicata *L*

44 in sequentibus: post [*N*] / in...apparebit: postea plenius patebit *AP[E]*

as a body whose base is the surface informed with the quality. This will be more fully clarified as we go along. Moreover, since in any kind of a body there is an infinite number of equivalent⁴ surfaces and the quality of any one of them is imagined as a body, it is not unfitting but necessary that one body be imagined to be at the same time in the place where another body—or even any other body whatever—is imagined to be. [We can think of this taking place] by penetration or by mathematical superposition⁵ or the simultaneous placing of the bodies so imagined. However, this penetration is not real. And although a surface quality is imagined by means of a body and it does not happen that a fourth dimension exists or is imagined,⁶ still a corporeal quality is imagined to have a double corporeity: a true one with respect to the extension of the subject in every dimension and another one that is only imagined from the intensity of this quality taken an infinite number of times and dependent upon the multitude of surfaces of the subject. The suitability of this imagined concept has been touched upon before and will be more fully apparent in what follows.

I.v On the figuration of qualities

Every linear quality is "figured" (i.e., represented in figures) by means of a surface perpendicularly erected upon a subject line. For let *AB* be a line informed with a quality [see Fig. 1]. And since by the preceding chapter this quality is designated by a surface, it is necessary that it be imagined as "figured" by the surface by which it is designated or imagined. The latitude of this surface designates the intensity of this quality. It is necessary also that any point of this surface or figure out-

⁴ The Latin text appears to have *superficies equales*. Either the *equales* ought to be deleted, or it is used with the meaning of surfaces that are equivalent or equal in thickness. One would suppose that Oresme would have conceived of them as being of infinitely small thickness, syncategorematically speaking, i.e.,

that they are thinner than any assignable quantity.

⁵ All manuscripts except *V* (which has *positionem*) have *suppositionem*. However, *superpositionem* makes much better sense and so I have rendered it such in my translation.

⁶ See the Commentary, I.iv, line 39.

I.v: BVAPFLD [ENMCGS]

1 Capitulum...qualitatum *BVLD[EC]* *om. A[M]* 5^m capitulum *P* De figuratione qualitatum capitulum 5^m *F[G]* Capitulum 5^m [*NS*] 5 c *mg. S* 6 *mg. A*

2 lignealis *A*

3 lineam *om. [S]* lignealem *A*

4 quoniam: quia *AP[EN]*

6 ipsa *om. LD[N]* / vel: aut *A* / vel ymaginatur *om. D*

7 intensionem...qualitatis: qualitatem intensionem *A* / istius¹ *om. ALPD[N]* / Oportet: et oportet *AP[E]* / etiam *om. AP[E]* igitur *G* / istius²: illius *AP* i^{us} [*E*] alicuius [*S*] / superficie *A*

[I.viii] Capitulum 8^m de qualitate triangulari rectangula

Omnis qualitas ymaginabilis per triangulum habentem rectum angulum super basim potest ymaginari per omnem triangulum habentem rectum angulum super eandem basim, et per nullam aliam figuram potest ymaginari. Quod enim aliqua qualitas sit ymaginabilis per talem triangulum patet ex capitulo precedenti eo quod aliqua potest esse proportionalis in intensione tali triangulo in altitudine; et illa est que vocatur communiter qualitas uniformiter difformis terminata ad non gradum, que tamen magis proprie potest dici qualitas uniformiter inequalis in intensione sicut triangulus cui ipsa proportionatur est altitudinis uniformiter inequalis. Similiter ipsa magis deberet dici terminari ad privationem quam ad non gradum. Sed quoniam alia locutio magis est apud modernos assueta et est satis transibilis, ideo in hoc tractatu eam recipio et admitto.

Quod vero predicta qualitas non possit per aliam figuram designari patet ex eo quod nulla figura alia a tali triangulo habente rectum angulum super basim est consimilis altitudinis seu proportionalis predicto triangulo, ut planum est intuenti; igitur nec proportionalis in altitudine isti qualitati in intensione. Ergo per eam non est ymaginabilis qualitas predicta, ut patet per 6^m capitulum. Sed quod indifferenter ista qualitas per omnem triangulum habentem rectum angulum super basim possit congrue ymaginari probatur sic.

Et sint duo trianguli tales, scilicet ABC minor et ABD maior, super basim AB ; deinde erigatur perpendiculariter linea EF in maiori triangulo, que secet lineam AC in puncto G [Fig. 4]. Quoniam ergo duo trianguli ABD et

I.viii: BVPFL

- 1 Capitulum...rectangula om. [SMA] capitulum 8^m [N] 8 mg. [SA] | triangulari VLP[G] trianguli BF[C] trian^u [E] | rectangula L[G] rectanguli VP rectanguli BF[C]
- 2 rectum om. P
- 4 figuram om. F
- 5 aliqua BV[SFMCG] talis LP[EAN] | per talem bis V
- 7 triangulo: triangulo rectangulo L[N] | illa: ista P[EA] | est que om. P que [A] | vocatur communiter tr. F[S] | qualitas tr. F[M] post est in linea 7
- 9 potest BVF[MNCG] posset LP[ESA] | in om. LP[NC] | intensione: intensive L[EN]
- 10 proportionaliter P

- 10-11 magis deberet tr. P[AG] magis debet [N]
- 11 dici om. L[SN]
- 12 magis est tr. VF[EMN] est [C] | transibilis BF[MN]
- 13 eam tr. P[EA] post ideo in linea 12
- 14 posset P[ESA]
- 15 eo quod: alio quia F[M] | tali om. P
- 16 seu BF[ESMAC] sive VLP[NG]
- 17 proportionalis tr. P[A] post altitudine | isti qualitati: istius qualitatis P
- 18 Ergo: igitur VLP[SG]
- 22 tales om. V[N] | scilicet om. F[M] | minor: brevior V
- 23 perpendiculariter linea tr. F[MC] | linea om. P[AN]
- 24 ergo: igitur V[SG]
- 24-25 ABD et AEF: ABG et AED F[M]

I.viii On a right-triangular quality

Every quality which is imaginable by a triangle having a right angle on the base can be imagined by every triangle having a right angle on the same base; and by no other figure can it be imagined. That some quality is imaginable by such a triangle is evident from the preceding chapter because some quality can be proportional in intensity to such a triangle in altitude. This quality is that which is commonly called a "uniformly difform quality terminated at no degree."¹ However, more properly it can be called a quality uniformly unequal in intensity just as the triangle to which it is proportional is uniformly unequal in altitude. Similarly it would be better to say that it is terminated at the "privation" [of the quality] rather than at "no degree." But since the other locution is the more customary one among moderns and represents the idea well enough, therefore I shall take it up and adopt it in this treatise.

That the aforesaid [uniformly difform] quality cannot be designated by any other figure is clear from the fact that no figure other than this kind of a triangle having a right angle on its base is of similar altitude or is proportional to the aforesaid triangle, as is plain to one who is observant. Therefore, no other figure is proportional in altitude to that quality in intensity, and hence the aforesaid quality is not imaginable by any other figure, as is clear by chapter six. But that this quality could be equivalently represented without any difference by every triangle having a right angle on the base is proved as follows.

On base AB let there be two such triangles, namely ABC the smaller one and ABD the larger one [see Fig. 4]. Then let line EF be erected perpendicularly in the larger triangle, cutting line AC in point G . Therefore, since the two triangles ABD

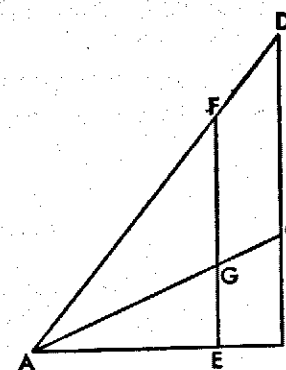


Fig. 4

Figure in MSS BLSGC. Letter C is missing in MS C.

I.viii

¹ I.e., "zero degree." I have retained "no degree" throughout to render *non gradum*.

Deo, de quo scriptum est in libro Danielis quod "ipse revelat profunda et abscondita, et novit in tenebris constituta."

Incipit secunda pars huius tractatus de difformitate successivorum

[II.i] Capitulum primum de duplici difformitate motus

9 Omnis motus successivus subiecti divisibilis habet partes et est divisibilis uno modo secundum divisionem et extensionem seu continuitatem mobilis, alio modo secundum divisibilitatem et durationem seu continuitatem temporis, tertio modo saltem ymaginative secundum gradus et intensionem velocitatis. A prima autem continuitate dicitur motus magnus vel parvus, a
10 secunda brevis aut longus, a tertia velox aut tardus. Habet itaque motus duplicem extensionem, unam subiectivam et aliam temporalem, et habet unam intensionem. Due autem extensiones possunt ymaginari quodam modo orthogonaliter seinvicem ad modum crucis intersecare, ita quod ex-
15 tensio durationis diceretur longitudo et extensio subiectiva vocaretur latitudo, intensio vero posset vocari altitudo ipsius motus seu velocitatis. Sed si iuxta premissa in 3^o capitulo prime partis intensio velocitatis appelleretur eius latitudo, tunc utraque extensionum ad intensionem comparata poterit dici longitudo et sic velocitas habebit duplicem longitudinem sicut habet duplicem extensionem, et in utraque istarum extensionum potest in-
20 tensio velocitatis multipliciter variari. Et quoniam difformitas oritur ex eo

28 *post Danielis add. [G] 3^o (I) capitulo*
28-29 *de ipse...constituta scr. mg. B in (?)*
Danl² 2 *et mg. A Daniel 28 (I) / et abs-*
condita om. B [P]

29 *et om. L / et...constituta om. [C] sed add.*
et sic est finis illius capituli et per conse-
quens totius tractatus / post constituta add.
[E] *Explicit prima pars huius tractatus*
deo gratias et add. [PFM] Explicit prima
pars huius operis et [A] explicit prima
pars et [N] Et sic finitur prima pars prin-

cipalis et [G] Explicit prima pars de inten-
sione qualitatum

Tit. et II. i: BVL

1-3 *Incipit (om. L)...successivorum BL[A]*
om. V[N] Incipit secunda [FMP] Secun-
da pars de figuratione et potentia successi-
vorum [G] Secunda pars particularis mg.
[C] *Incipit pars secunda [E] secunda pars*
mg. [S] Secunda pars [J]
4 *duplici om. L*

or by good or bad angels, or immediately by God, of whom it has been written in the book of Daniel² that "He revealeth deep and hidden things and knoweth what is in darkness."

Here begins the second part of this tract and it treats of the difformity of successive things

II.i On the double difformity of motion

Every successive motion of a divisible subject has parts and is divisible in one way according to the division and extension or continuity of the mobile, in another way according to the divisibility and duration or continuity of time, and in a third way—at least in imagination—according to the degree and intensity of velocity. From its first continuity motion is said to be "great" or "small"; from its second, "short" or "long," and from its third, "swift" or "slow." And so motion has two extensions, one that pertains to the subject and the other that pertains to time, and one intensity. Now the two extensions can be imagined in a certain way as mutually intersecting at right angles in the manner of a cross,¹ so that the extension of duration ought to be said to be "longitude" and the extension in subject ought to be called "latitude," while the intensity could be called the "altitude" of this motion or velocity. But according to what was premised in the third chapter of the first part, if intensity of velocity were to be called its "latitude," then each of the extensions in relationship to intensity could be called "longitude," and so velocity will have a double longitude just as it has a double extension, and in each of these extensions the intensity of velocity can be varied in multiple ways. And since dif-

² *Daniel 2:22.*

II.i

¹ *See the Commentary, II.i, lines 12-13.*

5 *subiecti: sive L*
6 *extensionem et divisionem L / continua-*
tionem V
7 *et: temporis et L*
7-8 *temporis: eiusdem L*
10 *aut^{1,2}: vel L*

12 *Due: que (?) L*
13 *orthogonaliter B[SG] / se-: seu L / seinvicem tr. V post crucis*
15 *posset BV[AFCM] potest L[ENSG]*
possit [P]
16 *intensio: in tempore [PFM]*

quod intensio varie extenditur, inde sequitur quod motus sive velocitas potest habere duplicem difformitatem vel etiam uniformitatem duplicem: unam secundum partes vel extensionem mobilis, que proprie dicitur uniformitas aut difformitas; aliam vero secundum partes vel durationem temporis, que proprie dicitur regularitas vel irregularitas. Habet ergo motus ratione subiecti uniformitatem vel difformitatem et ratione temporis sive secundum tempus regularitatem vel irregularitatem. Et secundum hoc diceretur quod motus celi est difformis et est regularis, motus vero gravis deorsum potest esse econtrario uniformis et irregularis et etiam potest esse uniformis et regularis vel difformis et irregularis. Sed non est possibile quod motus circularis sit uniformis. Verumptamen in sequendo modum loquendi consuetum vocabo quandoque improprie regularitatem nomine uniformitatis et irregularitatem nomine difformitatis, sed cum determinatione notificante concomitante vel prima.

[II.ii] Capitulum 2^m de tempore, quid sit et quod non est difforme

Primum omnium successivorum est tempus—ymmo tempus in una sui significatione est ipsamet successio morosa rerum mutabilium secundum prius et posterius seu earum duratio successiva. Nec tempus sic sumptum est motus sed est successio ipsius motus vel mobilis. Nam et si omnia quiescerent, adhuc esset tempus; aut si omnia mota moverentur velocius quam nunc moventur, non tamen velocitaretur tempus. Est itaque tempus secundum istam significationem quoddam accidens de predicamento quando. Nec est idem quod res temporalis, nec tamen est res separabilis a re temporali, quia sine tali re non potest absque implicatione contradictionis existere, nec ab ea potest separari etiam per divinam potentiam absolutam. Unde proprie loquendo tempus sic sumptum non est aliqua res sed est modus rei,

23 partes BV[ANG] partem L[EPFMCS]

24 aut: seu V / partem [PFMSC]

25 ergo B igitur LV

27-28 dicitur L

28 celi om. B

29 econtrario V[PS] eq^o BL[NG] econverso [E] eontra [AM] contrariis [F] contrarius [C] / post irregularis add. L[N] seu regularis et [E] seu et regularis / potest: posset V

29-30 et²... vel: sive [A] / uniformis... regularis: regularis et uniformis L[N] regularis

et difformis [E]

30 vel: et etiam [E] / vel...irregularis om. [F] (sed bab. M)

31 sequendo: sumpto B

32 improprie om. L

34 -ficante BV[AG] -ficante L -ficatione [ESG] -fi^{to} [PFNM] / concomitante vel VL[ENG] concomittante vel [A] concomitate vel B concomitantem vel [S] communicant cum [M] communicanter cum [PFC]

formity arises from the fact that intensity is variously extended, so it follows that motion or velocity can have double difformity or even double uniformity: one according to the parts or extension of the mobile—which is properly called “uniformity” or “difformity”—and the other according to the parts or duration of time—which is properly called “regularity” or “irregularity.”² Therefore, motion has “uniformity” or “difformity” by reason of subject, and “regularity” or “irregularity” by reason of, or according to, time. And according to this it ought to be said that the motion of the heavens is “difform” and “regular,” while the motion of a heavy body downward can be contrariwise “uniform” and “irregular;” and it also can be “uniform” and “regular,” or “difform” and “irregular.” But it is not possible for circular motion to be “uniform.” However, in following the customary manner of speaking, I shall sometimes improperly call “regularity” by the name of “uniformity” and “irregularity” by the name of “difformity,” but I shall do so with an accompanying or prior remark acknowledging this.³

II.ii On time: its nature and its non-difformity

The first of all successive things is time. In fact, time, in one signification of it, is itself the enduring succession¹ of mutable things according to before and after, or it is the successive duration of these things. But time assumed in this fashion is not the motion but is the succession of the motion or movable thing. For even if all things were at rest, still time would exist; or if all things in motion were moved more quickly than they are now moved, still the time would not be quickened. And so according to this signification time is a certain accident of the category “when.” Nor is time identical with a “temporal thing.” [But if not identical with a temporal thing] still it is not a thing separable from a temporal thing, for it could not exist in the absence of such a temporal thing without implying a contradiction.² Nor can it be separated from a temporal thing even by divine absolute power. Whence properly speaking time so assumed is not some *thing* but is rather a *mode*

² Ibid., lines 23-25.

³ Ibid., lines 33-34.

II.ii

¹ See the Commentary, II.ii, line 4.

² Ibid., lines 10-11.

II.ii: BVL

² est B[FA; cf. tab. cont.] sit VL[G]

³ ymmo tempus: tamen motus L cum tempus [E]

⁴ mutabilium: mobiliu L[E]

⁵ eorum L

⁷ mota om. V

significationem BV[FMPSG] acceptionem L[EN] acceptionem seu significationem [A] figurationem ? [C]

¹⁰ res² BV[FMAPCS] om. L[ENG]

quemlibet eorum moveri velocitate difformi, quoniam ex duabus vel pluribus
 5 velocitatibus uniformibus eiusdem corporis super varios polos aut circa
 centra diversa fit velocitas inequalis, irregularis, sive difformis. Unde etiam
 et stellas fixas dicunt astrologi moveri velocitate difformi. Oportet igitur
 10 quamlibet difformitatem celestis velocitatis secundum predicta aliquantulum
 figurari et opinandum est quod decenter figuretur et quod pulchre figure
 assimiletur. Nihilominus tamen aliqua astra aut omnia uno tempore moven-
 tur decentiori et pulchriori difformitate et velocitate magis consona quam
 alio tempore, acsi una vice facerent nobiliter tripudium quam alia sive
 15 meliorem concentum. Hunc enim concentum, ut ait Cassiodorus, "ratio
 tantum animo dedit sed auribus natura non prodidit." De quo etiam potest
 intelligi illud verbum Iob dicentis, "concentum celi quis dormire faciet?"
 quia sicut in Genesi legitur, "frigus et estus, hyems et estas, nox et dies, non
 requiescent." Mutatio autem huius concentus aut difformitatis potest esse
 20 una de causis quare aliquando hic inferius corpora celi immittunt influentias
 benigniores, aliquando minus benignas. Hec quoque causa potest alias causas
 veluti aspectus benivolos intendere vel remittere et malivolos, eo modo quo
 dicuntur malivoli intendere aut mitigare vel etiam totaliter impedire. Sed
 forsitan hec causa propter eius latentiam non fuit ab astrologis assignata.

[II.xiii] Capitulū 13^m de difformitate quorundam successivorum

Rerum quedam sunt ita successive quod non possunt aliquo modo per-
 manere, sicut tempus et motus. Alie sunt ita permanentes quod licet habeant
 esse vel durare temporaliter, divisibiliter, et successive, tamen earum es-
 5 sentia toto illo tempore eadem permanet nec potest esse aliquantulum successi-
 va, sicut substantie indivisibiles et immateriales. Sed prima illarum, que Deus
 est, nec habet essentiam successivam nec esse sive durare quoquomodo
 successivum. Ymmo indivisibiliter et infinite permanet per seipsam eterni-
 tate sua indivisibili et interminabili, que est idem quod ipsemet Deus. Alie

8 et om. L / igitur: ergo B
 9 difformitatem: velocitatem V
 10 opinandum: opus V / figuraretur V
 11 uno tempore tr. L[N]
 13 una vice: uno tempore L / alio L
 14-15 ratio tantum: non tamen L[C] non
 tantum [N] ideo tantum [FS]
 15 perdidit V
 16 Iob: Iob 38 S / faciet: sunt V
 17 legitur tr. L post sicut / hyems B[PG]

hiems L hyemps V[FN] hymens [S]
 yems [C] homines [A]
 18 requiescent V / huius om. L / aut diffor-
 mitatis om. L[N]
 19 immittunt LV[ANPS] immittant B[G]
 emittant [P] imittant ?[C] / ante influen-
 tias iter. V hic inferius
 21-22 vel...intendere om. L
 23 assignata: intenta L

certain that each of them is moved with a difform velocity since from two or more
 uniform velocities of the same body on various poles about diverse centers there
 arises an unequal, irregular, or difform velocity. Hence even the fixed stars, the
 astronomers say, are moved with difform velocity. Therefore, it is necessary that
 each difformity of celestial velocity be figured in some way according to what we
 have said, and one ought to think that it is figured properly and that it is assimilat-
 ed to a beautiful figure. Nevertheless some or all stars are moved in one time with a
 more fitting and more beautiful difformity and a more consonant velocity than in
 another time, as if to produce in one turn a nobler dance or better harmony than in
 another. For this harmony, as Cassiodorus says, "reason gave only to the mind,
 nature not producing it for the ears."¹ Of this harmony also can be understood the
 statement of Job: "who can make the harmony of heaven to sleep?"² For, as it is
 read in Genesis: "cold and heat, summer and winter, night and day shall not
 cease."³ But change of this harmony or difformity can be one of the causes why
 sometimes heavenly bodies emit below more benign, and at other times less benign,
 influences. This also can be the cause why [certain] other causes, such as favorable
 or unfavorable aspects, increase or decrease [things] in the way in which un-
 favorable ones are said to increase, or to mitigate, or even to impede [something]
 completely. But perhaps this cause was not assigned by astrologers because of the
 fact that it is hidden.

II.xiii On the difformity of certain successive things

Certain things are so successive that they cannot last in any way. Examples are
 time and motion. Other things are so permanent that, although they have to exist
 or last temporally, divisibly, and successively, still their essence remains the same
 during the whole time and it cannot in any way be successive. Examples are im-
 material and indivisible substances. But the first of these, which is God, neither
 has a successive essence nor does He exist or last in any way as a successive entity.
 In fact, He remains indivisibly and infinitely in His very self through His indivis-
 ible and indeterminable eternity, which is the same as God Himself. But there are

II.xii

¹ *Varias*, II, Letter 40 to Boethius (Ed. of Mommsen, 72, sect. 15). For Oresme's and other medieval views on the harmony of the spheres, see Zoubov, in *Mediaeval and Renais-*

sance Studies, Vol. 5 (1961), 98-100.

² Job 38:37.

³ Genesis 8:22. The Vulgate has "aestas et hiems."

II.xiii: BVL

3 sicut BL[PFMC] sicut sunt V[ANS G]

5 illo om. V

8-9 in eternitate V

10 vero sunt res quarum essentia est permanens, sed eadem vel similis potest
esse totaliter successiva. Cuiusmodi sunt quedam accidentia, sicut est propor-
tio, similitudo, curvitas, raritas, lumen, et universaliter omnis qualitas
15 intensibilis et remissibilis. Sicut enim in intensione curvitatibus vel raritatis
est continue alia et alia curvitas vel alia et alia raritas et in toto tempore illo
est una curvitas vel raritas successiva et conformiter in augmento proportio-
nis vel dissimilitudinis, ita ymaginor in intensione cuiuscunque qualitatis
intensibilis, sicut caliditatis vel albedinis, et similiter in eiusdem qualitatis
remissione. Nec est ibi realis multitudo sive superpositio graduum prout
aliqui opinantur. Talis ergo qualitas toto illo tempore in quo est alteratio
20 est una qualitas successiva et in qualibet eius parte est alia et alia, sed talis
qualitas est res permanens quotiens est in subiecto quod secundum ipsam
nullatenus alteratur. Sic igitur est quedam proportio permanens, alia succes-
siva; quedam dissimilitudo permanens, alia successiva; et sic de similibus
relationibus. Similiter quedam est curvitas permanens, alia successiva; que-
25 dam raritas permanens, alia successiva; quedam caliditas vel albedo perma-
nens, alia successiva; et sic universaliter de quibuslibet qualitibus inten-
sibilibus et remissibilibus vel etiam aliis accidentibus intensibilibus quibus-
cunque sicut de acutie angulari vel aliquo tali.

Alie autem sunt res quarum essentia est permanens. Et similis potest esse
30 successiva secundum partem non tamen secundum totum, sicut est illa forma
substantialis que est acquisibilis successive secundum partes quantitativas
subiecti et non secundum intensionem. Et ita opinor esse de omni forma
substantiali materiali; ita est etiam de angulo superficiali vel etiam corporeo.
Durante enim acquisitione vel deperditione talis forme ipsa continue est alia
35 et alia secundum partem non secundum totum. Et in toto illo tempore est
una res partialiter successiva, et motu tali non existente ipsa est permanens.
Nunc igitur in omnibus talibus sive totaliter sive partialiter successivis res
illa que sic est successiva secundum hoc quod in una parte temporis est
magis aut minus intensa quam in alia aut etiam maior vel minor uniformiter
40 et difformiter et diversimode secundum hoc ipsa iuxta ymaginationes prius
positas varie figuratur ad modum difformitatis ipsius velocitatis secundum
partes temporis, de qua dictum est ante, ita quod huiusmodi rei sic succes-

11 est om. L[N]

14 est continue tr. L[N]

15 vel: vel una L[PM]

16 vel: vel etiam L[N]

17 intensibilis: intensibilis et (vel CM) remis-
sibilis [NFMPC] / vel V[G] om.[FCMN] seu L sive [AS] et B / similiter:
sunt ?V

17-18 et...remissione om. [C]e contra [FMP]

17 qualitatis: caliditatis L

18 sive superpositio BV[AG] om. [CFMP]

sive suppositio L[N] sive sic positio [S]

19 ergo BL[ACSG] igitur V[PFMN]

20 sed: et V

23 dissimilitudo: similitudo V / alia: et alia
L

23-24 et...successiva om. B[G]

24 est om. V

26 et: et similiter L

27 et: vel L

other things whose essence is permanent while the same thing or something similar
can be totally successive. Of this sort are accidents such as ratio, similitude, cur-
vature, rareness, light, and, universally, every intensible and remissible quality. For
just as in the intensity of curvature or rarity there is continually different curvature
or different rarity while in the whole time it consists of one successive curvature or
rarity, and similarly in the cases of augmenting a ratio or a dissimilarity, so I imag-
ine it to be in the case of the intension of any intensible quality such as hotness or
whiteness, and similarly for the case of the remission of the same quality. Nor is
there in such cases real multitude or superposition of degrees, as some people
think.¹ Therefore, such a quality during the whole time in which there is alteration is
a successive quality and in any part of it it is different [from what it is in any other
part], but such a quality is a permanent thing whenever it is in a subject which is
not altered in any way with respect to this quality. Therefore, in the same way some
ratio is permanent while another is successive, some dissimilitude is permanent
while another is successive, and so on for similar relations. Similarly some cur-
vature is permanent while some is successive, some rareness is permanent while an-
other is successive, some hotness or whiteness is permanent while another is succes-
sive. And this is so universally for any intensible and remissible qualities at all or
even for any other intensible accidents such as [being] acute-angular or some such
thing.

But there are other things whose essence is permanent. And there can be a sim-
ilar thing whose essence is successive according to the part but not, however, ac-
cording to the whole. An example is that of a substantial form which can be ac-
quired successively according to the quantitative parts of the subject but not ac-
cording to intensity. And I think that this is so concerning every material sub-
stantial form and also concerning a surface or corporeal angle. For during the
acquisition or loss of such a form, it is continually different as to the part but not
as to the whole. And in the whole time it is a thing that is successive according to
part, and if such a motion does not exist then it is permanent. Now then in all such
things either totally or partially successive that thing which is thus successive, ac-
cording as it is more or less intense in one part of the time than in the other, or
also according as it is uniformly and difformly larger or smaller in different ways, is
figured with the previously posited imagined concepts in the manner of a difform-
ity of velocity with respect to the parts of time (of which we have spoken before),

II.xiii

¹ See the Commentary, II.xiii, lines 9-28.

30 tamen om. L[N]

33 est etiam tr. V[G] / etiam om. L[NSC]

34 continue est tr. L[N]

39-40 uniformiter et difformiter BV[ASG]
uniformitas sive difformitas L[NFMPC]

sive tempus est longitudo et intensio vel maioritas ipsius est sua latitudo. Et posset exemplificari de curvitate, de caliditate, de forma ignis, et sic de aliis similibus prius dictis.

Advertendum est tamen quod non est possibile quod res que est totaliter successiva sit uniformis, sicut proportio, curvitas, vel albedo, licet velocitas intensionis possit esse uniformis. Sed possibile est quod res partialiter successiva sit uniformis, sicut illud quod augetur quantitative ad unam partem et tantumdem precise diminuitur ad aliam. Si autem non diminueretur ad aliam partem vel si diminueretur non tantumdem precise, tunc illud esset difforme.

[II.xiv] Capitulum 14^m qualiter quorundam effectuum cause apparent ex predictis

Res itaque totaliter successiva modo predicto, sicut est curvitas vel albedo, cuius successio est propter illius rei intensionem vel remissionem, non potest esse successiva quin ipsa sit difformis. Verumptamen si talis res permaneret sine alteratione eius, ipsa posset dici secundum durationem temporis uniformis. Verbi gratia, si esset aliquod subiectum quod secundum curvitatē vel caliditatem intenderetur per unam horam et per secundam horam maneret in eodem statu sine alteratione et per tertiam intenderetur vel remitteretur, tunc in prima hora esset qualitas difformis, in secunda uniformis, et in tertia difformis, et totalis illa qualitas trium horarum esset secundum istam ymaginationem difformis difformitate composita. Cum igitur huiusmodi difformitas possit multipliciter variari et diversimode figurari secundum modos positos in prima parte huius capituli 13^o et 16^o, consequens est iuxta eiusdem partis capitula 22^m et 23^m et deinceps quod propter huiusmodi diversitatem configurationum diversificentur effectus et actiones naturalium qualitatum.

Verbi gratia, potest esse quod aliquotiens alicubi calor vel frigus seu quevis alia qualitas aeris variatur per tempus per intensionem et remissionem tali configuratione; propter quam ipsa est malorum humorum purgativa vel forsan generativa in corporibus humanis, vel in aliis, aut etiam inductiva alicuius pestifere qualitatis, vel etiam expulsiva, et sic de aliis; et

44 posset BVL[ASC] potest [PNFMG]

46 impossibile L

48 res: res que est L

50 tantumdem: nundum L / aliam: aliam partem L

51 diminueretur non tr. L

II.xiv: BVL

3 totaliter om. L[A]

8 caliditatem: qualitatem [PFM]

9 tertiam: tertiam horam L

11 istam om. L

12 difformitate: difformiter L / igitur: ergo V

so that the time of a successive thing of this sort is the longitude, and the intensity or largeness of it is its latitude. And it can be exemplified of curvature, of hotness, of the form of fire, and also of the other similar things previously mentioned.

It must be observed, however, that it is not possible for a thing which is wholly successive to be uniform, as is the case of ratio, curvature, or whiteness, although the velocity of intension could be uniform. But it is possible for something which is partially successive to be uniform, as is the case of that which is increased quantitatively in one part precisely in the same amount as it is diminished in another. But if it were not diminished in the other part or if it were not diminished precisely in the same amount, then that would be difform.

II.xiv How the causes of certain effects are evident from the aforesaid

Something that is wholly successive in the aforesaid way, like curvature or whiteness, and whose succession results from the intension or remission of the thing, cannot be successive unless it is difform. However, if such a thing were to last without alteration, it could be said to be uniform according to the duration of time. For example, if there were to be some subject which would be increased in intensity as to curvature or hotness for one hour and for the second hour would remain in the same state without alteration and for the third would be increased or decreased in intensity, then in the first hour it would be a difform quality, in the second a uniform quality, and in the third a difform quality, and the whole quality of the three hours would be, according to this imagery, a difform quality composite in difformity. Since, therefore, difformity of this sort could be multiply varied and figured in diverse ways according to the modes posited in chapters fifteen and sixteen of the first part of this work, it follows from chapters twenty-two, twenty-three, and thereafter, that because of the diversity of the configurations of this sort the effects and actions of natural qualities may be diversified.

For example, it can be that sometimes, somewhere, hot or cold or some other quality of the air is varied in such a configuration for a time by means of intension and remission, and that, accordingly, it is purgative or generative of bad humors in human or other bodies, or also that it is capable of inducing or expelling some pestiferous quality, and similarly with regard to other qualities. And this takes place

13 possit: potest L / variari om. L

15 et²: et sic L

19 per tempus per: post L

20 quam: quam et L

21 in² B[ANPFMSCG] om. V in etiam L

40 cordias domini in eternum cantabunt, quo cantico in gloriam gratie Christi
[...] nichil erit profecto illi iocundius civitati."

Incipit tertia pars
de acquisitione et mensura qualitatum
et velocitatum

[III.i] Capitulum primum: per quid ymaginanda est
acquisitio qualitatis

Duplici modo potest contingere successio in acquisitione qualitatis, scilicet secundum extensionem et secundum intensionem, sicut superius fuit dictum capitulo 4^o partis secunde. Acquisitio itaque extensiva qualitatis linearis ymaginanda est per motum puncti fluentis super ipsam lineam subiectivam, ita quod pars pertransita sit qualificata et pars nondum pertransita non qualificata. Sicut si punctus ϵ moveretur super lineam AB et quidquid esset ab eo pertransitum esset album et quidquid nondum esset pertransitum nondum esset album [Fig. 18(a)]. Acquisitio autem extensiva qualitatis superficialis ymaginanda est per motum linee dividantis partem superficiem

40 Christi om. L

41 profecto B[AFMSG] text. Aug. profecto L[VPC] perfectius [N] | illi iocundius tr. L[N] iocundius [A] | post civitati add. [PM] et sic est finis istius. Explicit secunda pars huius tractatus, et add. [F] et sic est finis, et add. [G] Explicit secunda pars, et add. [A] Amen. Explicit secunda pars huius tractatus, et add. [N] Explicit secunda pars et add. L Sequitur (I) pars huius tractatus, et add. [S] et sic explicit pars secunda

Tit. et III.i: BL

1-3 Incipit...velocitatum mg. BL[ANG] (except for the variants in BAL below) om. [VP] Incipit tertia etc. [M] Incipit 2^a (I) pars que est de figuratone et potentia qualitatum successivorum [F] Tertia pars

seu tractatus altius (alterius?) de acquisitione et mensura qualitatis et velocitatis [C]
1 3^a pars mg. [S], mg. [J] tertia pars tr. B tertia pars eiusdem [A]

2-3 qualitatis et velocitatis L

7 scilicet om. L[N]

8 capitulo 4^o [VANFMPSCG] tr. BL | partis secunde B[VANS] tr. L[G] prime partis [FMPC] | extensiva B[AFMSG] om. [C] extensive L[VN]

9 motum B[VN] (et cf. lin. 14, 15, 17-18, 22, 25, 30) modum L[MACSG] modum [P] medium [F]

9-10 subiectivam om. L subiectam [C]

10 nondum B[ANFMP] nondum L[VCSG] hic et alibi

12 esset¹ tr. L post pertransitum¹ / esset²: est L

14 superficialis: linearis L / est: esset L[N]

of God in eternity... certainly there will be no greater joy in that city than this song to the glory of the grace of Christ."⁵

Here begins the third part
[of this treatise]: On the Acquisition and Measure
of Qualities and Velocities

III.i How the acquisition of quality is to be imagined

Succession in the acquisition of quality can take place in two ways: (1) according to extension, (2) according to intensity, as was stated in the fourth chapter of the second part. And so extensive acquisition of a linear quality ought to be imagined by the motion of a point flowing over the subject line in such a way that the part

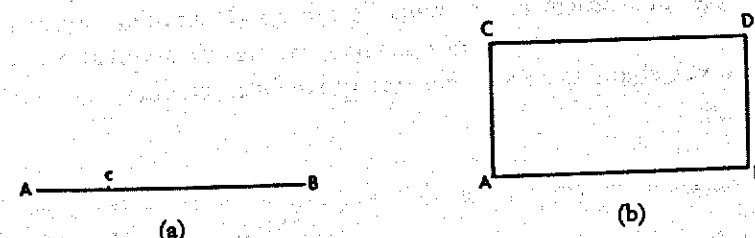


Fig. 18

Figure (a) in MSS BG. Figure (b) in MSS SJGL. MS L merely has two parallel lines AB and CD close together.

traversed has received the quality and the part not yet traversed has not received the quality. An example of this occurs if point ϵ were moved over line AB so that any part traversed by it would be white and any part not yet traversed would not yet be white [see Fig. 18(a)]. Further the extensive acquisition of a surface quality ought to be imagined by the motion of a line dividing that part of the surface that

⁵ De civitate Dei, XXII, 30 (ed. of Dombart, Vol. 2, 633-34): "Alioquin si se fuisse miseros nescituri sunt, quo modo, sicut ait psalmus, misericordias Domini in aeternum cantabunt? Quo

cantico in gloriam gratiae Christi, cuius sanguine liberati sumus, nihil erit profecto illi iocundus civitati." Augustine is referring to Psalms 88:2.

15 alteratam a parte nundum alterata. Et acquisitio extensiva corporee qualitatis conformiter ymaginanda est per motum superficiei dividensis partem alteratam a parte nundum alterata.

Acquisitio autem intensiva qualitatis punctualis ymaginanda est per motum puncti continue ascendentis super punctum subiectivum et motu suo describentis perpendicularem lineam ymaginatam super eundem punctum subiectivum. Acquisitio vero intensiva qualitatis linearis ymaginanda est per motum lineae perpendiculariter ascendentis super lineam subiectivam et suo fluxu vel ascensu derelinquentis superficiem per quam designatur qualitas acquisita. Verbi gratia [Fig. 18(b)]: Sit *AB* linea subiectiva. Dico igitur quod intensio puncti *A* ymaginatur per motum vel per ascensum perpendicularem puncti *C* et intensio lineae *AB* vel acquisitio intensivae ymaginatur per ascensum lineae *CD*. Acquisitio autem intensiva qualitatis superficialis conformiter ymaginanda est per ascensum superficiei motu suo ymaginato derelinquentis corpus per quod illa qualitas designatur. Et similiter acquisitio intensiva corporee qualitatis ymaginatur per motum superficiei quia superficies fluxu suo ymaginato derelinquit corpus et non contingit dare quartam dimensionem sicut dictum fuit 4^o capitulo prime partis.

Et sicut nunc dictum est de acquisitione qualitatis ita conformiter dicendum est et ymaginandum de deperditione, sive deperdatur extensio sive etiam intensio, ymaginatur enim talis deperditio per motus oppositos motibus prius dictis. Sicut etiam nunc dictum est de acquisitione aut deperditione qualitatis ita conformiter ymaginandum est de acquisitione aut deperditione velocitatis tam in extensione quam in intensione.

15 alteratam *om. L* alterate [FMP] / a: ab altera *L[N]*
 15-17 alterata....alterata *om. [C]* / Et...alterata *om. [FMP]*
 15-16 qualitatis *om. L[AN]*
 17 alteratam: alteram *L*
 19 subiectum *L[C]*
 19-21 et...subiectivum *om. L[N]*
 21 subiectivum *om. L* subiectum *[C]*
 22 subiectivam: summam *L*
 24 subiecta *L*
 25 per² *BL[FMP]* *om. [AVNSCG]* / ascensum *BL[VANS]* ascensionem [FMPCG]
 27 autem *om. L[N]* vero [F]
 29 designatur: ymaginatur [V]
 30 acquisitio *om. L[N]*
 30-31 superficiei: superficiei qualitatis *L[N]*
 32 fuit: est *L* / 4^o capitulo *tr. [VAC]*

34 deperditione *BL[VNSG]* perditione [AFMPC] / deperdatur: deperditio *L* / extensio *L[VNG]* etiam extensio *B* extensive [FMP] intensio [AS] intensive [C]
 35 etiam intensio *B[V]* *om. L* intensio [NG] etiam intensive [F] etiam extensive [C] etiam extensio [A] intensive [MP] extensio [S]
 36 etiam *om. [G]* enim *L[N]*
 36-37 aut deperditione *BL[V]* *om. [NFMP]* *C* de perditione [A] et de perditione [G] perditione [S]
 37-38 deperditione: de perditione [AFP] perditione [V]
 38 in extensione quam in intensione *B[VFM]* *PG* in (*om. C*) intensione quam in (*om. S*) extensione *L[SC]* in intensum quam extensum [N] extensive quam intensive [A]

has been altered from the part not yet altered. And the extensive acquisition of a corporeal quality in a similar way is to be imagined by the motion of the surface dividing the part altered from the part not yet altered.¹

The intensive acquisition of punctual quality is to be imagined by the motion of a point continually ascending over a subject point and by its motion describing a perpendicular line imagined [as erected] on that same subject point. But the intensive acquisition of a linear quality is to be imagined by the motion of a line perpendicularly ascending over the subject line and in its flux or ascent leaving behind a surface by which the acquired quality is designated. For example [see Fig. 18(b)], let *AB* be the subject line. I say, therefore, that the intensio of point *A* is imagined by the motion, or by the perpendicular ascent, of point *C*, and the intensio of line *AB*, or the acquisition of the intensity, is imagined by the ascent of line *CD*. Further, the intensive acquisition of a surface quality is in a similar way to be imagined by the ascent of a surface, which (by its motion) leaves behind a body by means of which that quality is designated. And similarly the intensive acquisition of a corporeal quality is imagined by the motion of a surface because a surface by its imagined flux leaves behind a body, and one does not have to pose a fourth dimension, as has been said in the fourth chapter of the first part.

One should speak and conceive of the loss of quality in the same way that we have now spoken of its acquisition, whether that loss is of extension or intensity. For such loss is imagined by movements which are the opposite of the movements described before. Furthermore, one ought to speak of the acquisition or loss of velocity, both in extension and intensity, in the same way we have just spoken of the acquisition or loss of quality.

III.i

¹ See the Commentary, III.i, lines 13-17.

ymaginetur ens successivum. Unde in Ysaia dicitur, "erit lux lune sicut lux solis, et lux solis erit septemplex sicut lux septem dierum," quia videlicet
 30 lux unius diei septemplex intensa equalis est luci que per septem dierum spatium extenderetur.

[III.vii] Capitulum 7^m de mensura qualitatum et velocitatum
 difformium

Omnis qualitas, si fuerit uniformiter difformis, ipsa est tanta quanta foret qualitas eiusdem subiecti vel equalis uniformis secundum gradum
 5 puncti medii eiusdem subiecti; et hoc intelligo si qualitas fuerit linearis. Et si fuerit superficialis, secundum gradum linee medie; si vero fuerit corporalis, secundum gradum medie superficiiei, semper conformiter intelligendo. Istud ostenditur primo de lineari. Sit igitur una qualitas ymaginabilis per
 10 triangulum *ABC* que est uniformiter difformis terminata ad non gradum in puncto *B* [Fig. 21(a)]; et sit *D* punctus medius linee subiective, cuius quidem puncti gradus vel intensio ymaginatur per lineam *DE*. Igitur qualitas que esset uniformis per totum subiectum secundum gradum *DE* ymaginabilis est per quadrangulum *AFGB*, ut patet per 10^m capitulum prime partis. Constat autem per 26^m primi Euclidis quod duo parvi trianguli *EFC* et *EGB* sunt

28 ens *B[VAFMPC]* esse *L[SG]* esse ens
 [N]

29-30 videlicet lux tr. *L* lux [N]

31 extenditur *L[S]*

III.vii: *BL*

3-4 ipsa...uniformis om. [FMP]

5 si...linearis: qualitate lineari *B*

7 medie superficiiei tr. *L[NS]* superficiiei [V]
 / semper: secundum hoc *L*

8 ostenditur primo tr. *L[G]* ostenditur
 [FMPC]

11-12 esset *B[VASG]* est *L[NFMPC]*

13-14 Constat autem *B[VSG]* constatque
L[N] constat et [FMPC]

14 *EFC*: *EFG B[V]*

if it is imagined to be a successive entity. Whence it is said in Isaias:¹ "And the light of the moon shall be as the light of the sun, and the light of the sun shall be sevenfold as the light of seven days," for evidently the light of one day increased intensively by sevenfold is as the light which would be extended through a space of seven days.

III.vii On the measure of difform qualities and
 velocities

Every quality, if it is uniformly difform, is of the same quantity as would be the quality of the same or equal subject that is uniform according to the degree of the middle point of the same subject.² I understand this to hold if the quality is linear. If it is a surface quality, [then its quantity is equal to that of a quality of the same subject which is uniform] according to the degree of the middle line; if corporeal, according to the degree of the middle surface, always understanding [these concepts] in a conformable way. This will be demonstrated first for a linear quality. Hence let there be a quality imaginable by *ABC*, the quality being uniformly difform and terminated at no degree in point *B* [see Fig. 21(a)]. And let *D* be the middle point of the subject line. The degree of this point, or its intensity, is imag-

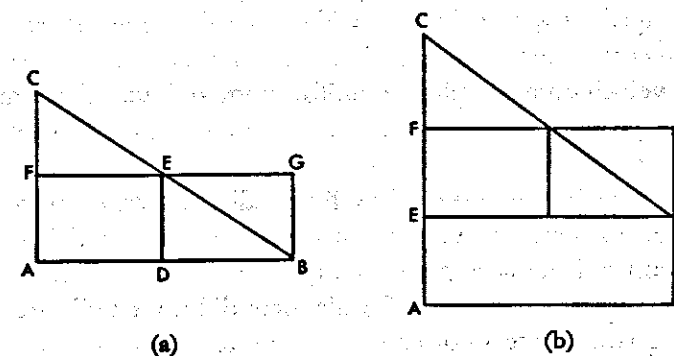


Fig. 21

Figures in *BLSJG*. Figures are rotated through 90° in MS *G*. In figure (b) in MS *L*, there is no center perpendicular. In MS *J*, line *ED* is missing and the center perpendicular is marked *KH*. Both figures are reversed in MS *J*.

ined by line *DE*. Therefore, the quality which would be uniform throughout the whole subject at degree *DE* is imaginable by rectangle *AFGB*, as is evident by the tenth chapter of the first part. Therefore, it is evident by the 26th [proposition] of [Book] I [of the *Elements*] of Euclid² that the two small triangles *EFC* and *EGB*

III.vii

¹ Isaias 30:26.

² See the Commentary, III.vii, lines 3-5.

³ *Ibid.*, line 14.

15 equales. Ergo maior triangulus *BAC* qui designat qualitatem uniformiter difformem et quadrangulus *AFGB* qui designaret qualitatem uniformem secundum gradum puncti medii sunt equales. Ergo qualitates per huiusmodi triangulum et quadrangulum ymaginabiles sunt equales. Et hoc est propositum.

20 Eodem modo potest argui de qualitate uniformiter difformi terminata utrinque ad certum gradum, sicut esset qualitas ymaginabilis per quadrangulum *ABCD* [Fig. 21(b)]. Protrahatur enim linea *DE* equedistans basi subiecte et fieret triangulus *CED*. Deinde protrahatur per gradum puncti medii linea *FG* equalis et equedistans basi subiecte, et protrahatur etiam linea
25 *GD*. Tunc sicut prius probabitur quod triangulus *CED* et quadrangulus *EFGD* sunt equales. Ergo addito utrobique quadrangulo communi *AEDB* fient duo tota equalia, scilicet quadrangulus *ACDB* qui designat qualitatem uniformiter difformem et quadrangulus *AFGB* qui designaret qualitatem uniformem secundum gradum puncti medii ipsius subiecti *AB*. Igitur
30 per capitulum 10^m prime partis qualitates per huiusmodi quadrangulos designabiles sunt equales.

Conformiter potest argui de qualitate superficiali ac etiam de corporali. De velocitate vero omnino dicendum est sicut de qualitate lineari, dum
35 tamen loco puncti medii capiatur instans medium temporis velocitatem huiusmodi mensurantis. Sic itaque patet cui qualitati aut velocitati uniformi adequatur qualitas sive velocitas uniformiter difformis. Proportio autem qualitatum et velocitatum uniformiter difformium est sicut proportio qualitatum et velocitatum simpliciter uniformium quibus adequantur. Et de mensura et proportionem illarum uniformium dictum est in capitulo precedenti.
40

Si autem qualitas seu velocitas fuerit difformiter difformis, tunc, si componatur ex partibus uniformibus aut uniformiter difformibus, ipsa poterit mensurari per suas partes, de quarum mensura dictum est ante. Si vero qualitas fuerit alio modo difformis, sicut difformitate illa que per curvitatem designatur, tunc oporteret recurrere ad mensurationem figurarum
45 curvarum inter se aut earum cum rectis figuris; et hoc est alterius speculationis. Sufficiant ergo que dicta sunt.

21 utrinque *B[SG]* om. [*C*] utriusque *L[A]*
utrobique [*VN*] uterque [*FMP*]

23-24 et...et² om. *L*

26 sunt *B[VANS]* fient *L* erunt [*FMPC*]

26-27 equales...fient *B[VANS]* om. [*FMPC*] equales quare *L*

27-28 *ACDB*...quadrangulus om. *L*

28 designaret *B[AVS]* designat *L[FNM]*
?*P*, ?*C*, ?*G*]

33 De¹ om. *L[N]*

34 capiatur *B[VSG]* om. [*FMP*], tr. *L[AN]*
post temporis / instans: instantis *L* infert
[*FMP*]

36 qualitatis aut velocitatis *L*

37 et: aut *L[N]* sive [*A*]

38 Et om. *L[AN]*

39 et: et de *L[VMP]* / uniformium om. *L[N]*

41 qualitas...velocitas: velocitas seu quali-

are equal. Therefore, the larger *ABAC*, which designates the uniformly difform quality, and the rectangle *AFGB*, which designates the quality uniform in the degree of the middle point, are equal. Therefore the qualities imaginable by a triangle and a rectangle of this kind are equal. And this is what has been proposed.

In the same way it can be argued for a quality uniformly difform terminated in both extremes at a certain degree, as would be the quality imaginable by quadrangle *ABCD* [see Fig. 21(b)]. For let line *DE* be drawn parallel to the subject base and *ACED* would be formed. Then let line *FG* be drawn through the degree of the middle point which is equal and parallel to the subject base. Also, let line *GD* be drawn. Then, as before, it will be proved that *ACED* = *EFGD*. Therefore, with the common rectangle *AEDB* added to both of them, the two total areas are equal, namely quadrangle *ACDB*, which designates the uniformly difform quality, and the rectangle *AFGB*, which would designate the quality uniform at the degree of the middle point of the subject *AB*. Therefore, by chapter ten of the first part, the qualities designatable by quadrangles of this kind are equal.

It can be argued in the same way regarding a surface quality and also regarding a corporeal quality. Now one should speak of velocity in completely the same fashion as linear quality, so long as the middle instant of the time measuring a velocity of this kind is taken in place of the middle point [of the subject].³ And so it is clear to which uniform quality or velocity a quality or velocity uniformly difform is equated. Moreover, the ratio of uniformly difform qualities and velocities is as the ratio of the simply uniform qualities or velocities to which they are equated. And we have spoken of the measure and ratio of these uniform [qualities and velocities] in the preceding chapter.

Further, if a quality or velocity is difformly difform, and if it is composed of uniform or uniformly difform parts, it can be measured by its parts, whose measure has been discussed before. Now, if the quality is difform in some other way, e.g. with the difformity designated by a curve, then it is necessary to have recourse to the mutual mensuration of the curved figures, or to [the mensuration of] these [curved figures] with rectilinear figures; and this is another kind of speculation.⁴ Therefore what has been stated is sufficient.

³ *Ibid.*, lines 33-35.

⁴ *Ibid.*, line 46.

tas *L[N]* difformitas sive qualitas [*A*]
44 sicut om. [*C*] sicut de *L[N]*

47 Sufficiant...sunt *BL[VS]* om.
[*ANFMPCG*]

[III.viii] Capitulum 8^m de mensura et intensione in infinitum
quarundam difformitatum

Superficies finita potest fieri quantumlibet longa vel alta per variationem extensionis absque eius augmento. Nam talis superficies habet longitudinem et latitudinem et possibile est ipsam secundum unam dimensionem quantumlibet augeri ipsa tamen non augmentata simpliciter dummodo secundum aliam dimensionem proportionaliter minuatur, et ita est etiam de corpore. Verbi gratia de superficie [Fig. 22]: accipiaturs superficies quadrata pedalis, cuius basis sit linea *AB*; et sit alia superficies similis et equalis, cuius basis sit

III.viii: BL

1 intensione BL[V] extensione [FCG]

6 tamen om. L[ANS]

7 proportionabiliter V / diminuaturs [VAN C] / ita est etiam B[VS] ita est [FMP C] ita etiam L[ANG]

III.viii On the measure and intension to infinity of
certain difformities

A finite surface can be made as long as we wish, or as high, by varying the extension without increasing the size. For such a surface has both length and

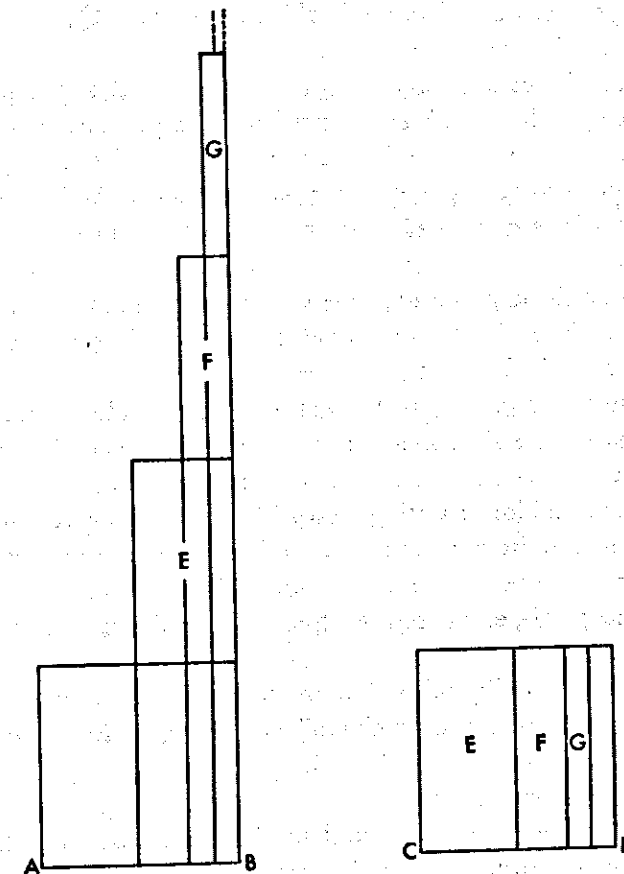


Fig. 22

Figures in MSS BLJG. MS L reverses the left drawing and adds another "foot" to the left of E on the right drawing. MS B makes the base of the left drawing CD and that of the right drawing AB. MS G reverses both drawings.

breadth and it is possible for it to be increased in one dimension as much as we like without the whole surface being absolutely increased so long as the other dimension is diminished proportionally, and this is also true of a body. For example¹ [see Fig. 22], in the case of a surface, let there be a surface of one square foot in area whose base line is *AB*; and let there be another surface, similar and equal to it,

III.viii

¹ See the Commentary, III.viii, lines 8-28.

10 linea *CD*, que ymaginetur in infinitum dividi per partes continue proportionales secundum proportionem duplam super basim *CD* eodem modo divisam, et sit *E* prima pars, et *F* secunda, et *G* tertia, et sic de aliis. Sumatur igitur prima istarum partium, scilicet *E*, que est medietas sui totius, et ponatur super primam superficiem versus extremum *B*. Deinde super
 15 totum hoc ponatur secunda pars, scilicet *F*, et iterum super totum hoc ponatur tertia pars, scilicet *G*, et ita de aliis in infinitum. Quo facto, ymaginetur basis *AB* dividi per partes continue proportionales secundum proportionem duplam, eundo versus *B*. Et statim patebit quod super primam partem proportionalem lineae *AB* stat superficies alta per unum pedem, et super
 20 secundam partem stat superficies alta per duos pedes, et super tertiam per tres et super quartam per quatuor, et sic ulterius in infinitum, et tamen totalis superficies non sit nisi duo pedalia prius data in nullo augmentata. Et per consequens totalis superficies que stat super lineam *AB* est precise quadrupla ad illam sui partem que stat super primam partem proportionalem eiusdem lineae *AB*. Illa ergo qualitas sive velocitas que proportionaretur
 25 in intensione huic figure in altitudine esset precise quadrupla ad partem sui que foret in prima parte temporis vel subiecti secundum huiusmodi divisionem. Verbi gratia: sit prima pars proportionalis secundum proportionem duplam ipsius lineae *AB* versus extremum *A* alba seu calida aliquantum et
 30 secunda duplo albior et tertia triplo albior et quarta quadruplo et sic consequenter in infinitum utrinque secundum seriem numerorum. Tunc ex predictis apparet quod totalis albedo lineae *AB* est precise quadrupla ad albedinem prime partis, et ita esset de albedine superficiali, ac etiam de corporali si conformiter esset intensa.

35 Eodem modo si aliquod mobile moveretur in prima parte proportionali alicuius temporis taliter divisi aliquali velocitate, et in secunda moveretur duplo velocius, et in tripla triplo, et in quarta quadruplo, et sic consequenter in infinitum semper intendendo, velocitas totalis esset precise quadrupla ad velocitatem prime partis, ita quod illud mobile in tota hora pertransiret
 40 precise quadruplum ad illud quod pertransivit in prima medietate illius hore; ut si in prima medietate vel parte proportionali pertransiret unum pedem, in toto residuo pertransiret tres pedes et in toto tempore pertransiret quatuor pedes. Istud autem alias demonstravi demonstratione subtiliori et

19-20 superficies...stat om. [FMPC]
 20 partem om. *B* / stat...alta om. *L*[*N*] / pedes: partes *L* / tertiam: tertiam partem *L*[*AN*]
 21 tres: tres pedes *L*[*AN*]
 22-23 totalis....consequens om. *L*
 24 sui partem om. *L* superficiem *V*
 25-29 Illa....*AB* bis *L*
 27-28 divisionem: dimensionem *L*[*AN*]
 28 pars om. *B*[*V*]

29 *A* om. [*V*] *DL* / aliquantum: aliquantulum *L*[*SC*]
 30 quarta: quarta in *L*[*ANS*]
 31 utrinque *B*[*FMPG*] utrobique [*VC*] utriusque *L*[*AN*] utrinam ?[*S*]
 32 apparet *B*[*VFMSC*] patet *L*[*ANPC*]
 33 ita om. *B* / ac etiam *B*[*SG*] et etiam [*V*] et *L*[*ANFMP*]
 35 moveretur: movetur *B* moveatur [*A*]

whose base line is *CD*. Let the latter surface be imagined to be divided on line *CD* to infinity into parts continually proportional according to the ratio 2 to 1, with its base divided in the same way. Let *E* be the first part, *F* the second, *G* the third, and so on for the other parts. Therefore, let the first of these parts, namely *E*, which is one half the whole surface, be taken and placed on top of the first surface towards the extremity *B*. Then upon this whole let the second part, namely *F*, be placed, and again upon the whole let the third part, namely *G*, be placed, and so on for the others to infinity. When this has been done, let base line *AB* be imagined as being divided into parts continually proportional according to the ratio of 2 to 1, proceeding towards *B*. And it will be immediately evident that on the first proportional part of line *AB* there stands a surface one foot high, on the second a surface two feet high, on the third one three feet high, on the fourth four feet high, and so on to infinity, and yet the whole surface is only the two [square] feet [in area] previously given, without augmentation. And consequently the whole surface standing on line *AB* is precisely four times its part standing on the first proportional part of the same line *AB*. Therefore, that quality or velocity which would be proportional in intensity to this figure in altitude would be precisely four times the part of it which would be in the first part of the time or the subject so divided. For example, let the first part (towards extreme *A*) of the proportional parts divided along *AB* according to the ratio 2 to 1 be a certain amount white or hot, the second twice as white [intensively], the third three times as white, the fourth four times, and so on to infinity on both sides according to the [natural] series of [whole] numbers. Then from the prior statements it is apparent that the total whiteness of line *AB* is precisely four times the whiteness of the first part; and it would be the same for a surface, or for a corporeal, whiteness, if it were increased in intensity in similar fashion.

In the same way, if some mobile were moved with a certain velocity in the first proportional part of some period of time, divided in such a way, and in the second part it were moved twice as rapidly, and in the third three times as fast, in the fourth four times, and increasing in this way successively to infinity, the total velocity would be precisely four times the velocity of the first part, so that the mobile in the whole hour would traverse precisely four times what it traversed in the first half of the hour; e.g., if in the first half or first proportional part it would traverse one foot, in the whole remaining period it would traverse three feet and in the total time it would traverse four feet. Moreover I have demonstrated this elsewhere with a more subtle and difficult demonstration.² But this present treatment con-

² *Ibid.*, lines 35-44.

41 ut: et *L*[*AN*] / pertransiret: transivit *L*[*AN*]

42 pertransiret²: transiret [*AN*]

43-45 Istud...pretermitto om. [*G*]
 43 demonstravi: demonstratur *L*

difficiliori. Sed ista est magis conformis huic tractatui et sufficit; ideo aliam
45 premitto.

[III.ix] Capitulum 9^m de quodam alio exemplo

Ymagnetur linea *AB* [Fig. 23] in infinitum divisam signative per partes
continue proportionales secundum proportionem quadruplam, ita quod
prima pars proportionalis sit tres quarte totius et secunda sit tres quarte resi-
5 dui primi et tertia sit tres quarte residui secundi et sic consequenter in in-
finitum; ut si tota linea esset sicut 64, tunc prima pars proportionalis
secundum istam divisionem esset 48 et secunda esset 12 et tertia esset 3 et
quarta esset tres quarte residui, scilicet unius, et sic ultra. Hoc posito,
super primam istarum partium lineae *AB* ymagnetur superficies aliquante
10 altitudinis uniformis, et super secundam partem sit superficies duplo altior,
et super tertiam quadruplo altior, et super quartam octuplo altior, et sic in
infinitum semper duplando, ita quod altitudines superficierum procedant
continue augmentando secundum proportionem duplam et longitudines
continue diminuendo secundum proportionem subquadruplam. Et sic secun-
15 da pars superficialis est ad primam subquadrupla in longitudine et dupla in
altitudine, et ita se habet tertia ad secundam et quarta ad tertiam, et sic ulter-
ius procedendo. Dico ergo quod totalis superficies vel figura est precise
dupla ad primam sui partem, i.e., ad partem que est super primam partem
proportionalem lineae *AB*.

20 Quod probatur sic: Sit superficies que est super primam partem sicut 48.
Igitur illa superficies que est super secundam partem, si non esset altior
quam prima, ipsa esset sicut 12. Nunc autem ipsa est duplo altior quam
prima. Igitur ipsa secunda est sicut 24, et per idem tertia est sicut 12, et
quarta sicut 6, et sic ulterius procedendo secundum proportionalitatem
25 subduplam. Unde et ista proportio istarum superficierum posset ostendi per
ea que dicta sunt 6^o capitulo huius partis. Habemus igitur quod prima pars
est dupla ad secundam et secunda dupla ad tertiam et tertia dupla ad quar-
tam, et sic consequenter. Ergo totius aggregati ex omnibus infinitis prima
pars est medietas, et secunda est medietas residui, et sic ultra. Ergo totum

III.ix: *BL* [M has not been used here; it is illegible.]

2 ymaginemur lineam *L* [*AN*]

5 et...secundi om. [*AN*] / secundi om. [*C*]
terti secundi *L*

6 tunc *B* [*VFPS*] om. *L* [*ANCG*]

8 scilicet om. *B* [*C*]

9 illarum *L* [*NCG*]

12 duplando: duplicando *L* subduplando [*A*]
subduplicando [*N*] / superficierum: super-
ficierum partium *L* [*AN*]

14 subquadruplam: quadruplam *L*

15 et dupla: duplaque *L* [*AN*]

19-20 proportionalem....sicut om. *L*

21 partem om. *L*

22-23 Nunc....24 om. *L*

forms more to [the scope of] this treatise, and suffices. Therefore I shall pass over
the other [demonstration].

III.ix Another example of the same¹

Let there be imagined a line *AB* [see Fig. 23] divided infinitely into desig-
nated parts continually proportional according to the ratio of 4 to 1, so that the
first proportional part is three-fourths of the whole, the second is three-fourths of
the first remainder, the third is three-fourths of the second remainder and so on
successively to infinity. Thus if the whole line were as 64, then the first propor-
tional part, according to this division, would be 48, the second would be 12, the third
would be 3, and the fourth would be three-fourths of the remainder, i.e., $\frac{3}{4}$ of 1,
and so on. With this posited, let a surface of some uniform altitude be imagined as
standing on the first part of line *AB*, and on the second part a surface twice as
high, on the third one four times as high, on the fourth one eight times as high, and
so on to infinity, always doubling the height, so that the altitude proceeds to
increase continually according to the ratio of 2 to 1 and the base length is continual-
ly diminished according to the ratio of 1 to 4. And so the surface of the second
part is related to that of the first by being one-fourth of it in length and twice it in
height; the third is similarly related to the second, and the fourth to the third, and
proceeding thus to the end. I say, therefore, that the whole surface or figure is
precisely twice the first part, i.e. twice the part which stands over the first propor-
tional part of line *AB*.

Proof: Let the surface over the first part be as 48. Therefore the surface over the
second part, if it were no higher than the first, would be as 12. However, it is
twice as high as the first. Therefore the second surface is [actually] as 24. By the
same reasoning, the third surface is as 12, and the fourth is as 6, proceeding thus
to the end, always by a ratio of 1 to 2. Hence it is that this ratio of these surfaces
could be demonstrated by what has been said in the sixth chapter of this part.
Therefore, we have it that the first part is double the second, the second double the
third, the third double the fourth, and so on successively. Therefore, the first part
is half the whole aggregate of all the infinite parts, the second is half of the re-
mainder, and so on. Therefore, the whole aggregate is precisely double its first

III.ix

¹ See the Commentary, III.ix, lines 2-40.

23 12 corr. ex 6 in omnibus MSS (sed diag. J)

babet 6)

babet 12)

25 ista: istarum *L* ipsa [*SG*]

24 6 corr. ex. 3 in omnibus MSS (sed diag. J)

Quis formarum latitudines multipliciter variantur que
multiplicitas difficulter discernitur, nisi ad figuras geo-
metricas consideratio referatur. Ideo premissis quibusdam
divisionibus latitudinum cum diffinitionibus suis, infinitas
species earundem ad infinitas figurarum species applicabo;
ex quibus propositum clarius apparebit.

(Prima divisio.) Latitudinum quedam uniformis, quedam
difformis. Latitudo uniformis est que est eiusdem gradus
per totum. Latitudo difformis est que non est eiusdem gradus
per totum.

(2^a divisio.) Latitudinum difformium, quedam est secundum se
totam difformis, quedam non secundum se totam difformis.

Latitudo secundum se totam difformis est cuius nulla pars est
uniformis. Latitudo que non est secundum se totam difformis
est illa cuius aliqua pars est uniformis. Unde simul stant
quod latitudo sit difformis cuius aliqua eius pars sit
uniformis sicut in secunda parte melius sperebit.

(3^a divisio.) Latitudinum secundum se totas difformium, quedam
est uniformiter difformis, quedam difformiter difformis.

Latitudo uniformiter difformis est illa cuius est equalis ex-
cessus graduum inter se equaliter distantium. Latitudo dif-
formiter difformis sumitur per oppositum, videlicet, cuius non
est equalis excessus graduum inter se eque distantium.

Because the latitudes of forms are varied in many ways, this
multiplicity is discerned with difficulty unless reference is made
to geometric figures. Accordingly, having laid out certain divisions
of latitudes with their definitions, I shall apply the infinite kinds
of these (latitudes) to the infinite kinds of figures; from these
the subject will appear more clear.

(First division.) Of latitudes, some are uniform, some difform.
A uniform latitude is that which is of the same degree throughout.
A difform latitude is that which is not of the same degree through-
out.

The second division is that: of difform latitudes, some are
difform throughout, some are not difform throughout. A latitude
difform throughout is that of which no part is uniform. A latitude
which is not difform throughout is that of which some part is uniform.
Whence it is evident that there may be a difform latitude, of which
some part may simultaneously be uniform, as in the second part will
be made more apparent.

The third division is that: of latitudes difform throughout,
some are uniformly difform, some difformly difform. A latitude uni-
formly difform is that in which there is an equal increment of degrees
that are equidistant among themselves. A latitude difformly difform
is defined in the opposite manner, namely, (it is that) in which there
is not an equal increment of degrees that are equidistant among
themselves.

7. Anonym: *Latitudines breves* 1, 2 and 3.

From: Daniel A. Di Liscia, "The *Latitudines breves* and Late Medieval University Teaching", *SCIAMVS* 17 (2016), pp. 55-120.

Latitudines breves 1

⟨Sequitur⟩¹ liber latitudinum formarum, qui subordinatur phylosophie speculative medie inter physicam et geometriam, secundum aliquos quia "latitudo" est terminus geometrie et "forme" terminus physice. Utilitas istius scientie est ad intelligendum multas partes physice ad alterationem, an fiat secundum intensum vel remissum, et ad materiam *tertii Physicorum*. Et nota: gradualis intensio forme dicitur eius latitudo, et sic intensio gradualis et latitudo gradualis supponunt pro eadem et similiter latitudo forme. Non tamen sinonime, quia ly latitudo forme non est absolute intensio gradualis, sed connotat relative talem intensionem esse ymaginabilem per figuram geometricam et extensam.

Et non solum forma est latitudinabilis quo ad eius intensionem sed etiam motus localis et forma substantialis quo ad fieri vel mutari. Consequenter intensio gradualis forme vocatur eius latitudo; eius vero extensio quo ad subiectum vel quo ad durationem vocatur longitudo. Extensio mensuratur per lineam in subiecto, latitudo autem aut intensio per lineam perpendiculariter super eandem lineam erectam. Linea non perpendiculariter data aut alia existens non est latitudo acquisita quo ad subiectum et sit fuerit acquisita, tunc illomodo, quia alias esset latitudo sine longitudine, quod est impossibile. Et sic arguitur longitudo notior est ibi quam latitudo quia facilius scimus extensionem rei quam intensionem.²

It follows the book of the latitude of the forms, which is subordinated to the middle speculative philosophy between physics and geometry, according to some <people> because "latitude" is a term of geometry and "form" a term of physics. The utility of that science is for understanding many parts of the physics referred to alteration, whether it takes places with regard to the intension or remission, and for the matter of the third book of the *Physics*. And note: the gradual intension of form is called its latitude so that "gradual intension" and "gradual latitude" suppose for the same, and similarly "latitude of form". However, not as synonyms, because "latitude of form" is not absolutely "gradual intension" but it connotes relatively that a such intension is to be represented by a geometric and extended figure.

And not only the form is latitudinable according to its intension but also the local motion and the substantial form with respect to its coming-into-being and being mutated. Consequently, the gradual intension of the form is called its latitude; its extension, however, with respect to the subject or with respect to the duration, is called longitude. The extension is measured by a line in the subject; the latitude, however, or the intension, by a line erected perpendicularly on the same line. A line which is not perpendicularly given [=represented] or which is in any other way existent, is not a latitude acquired with respect to the subject and, if it would be acquired, then in that <before mentioned> way, since otherwise there would be a latitude without longitude, which is impossible. And so, it is argued that the longitude is better known than the latitude, because we recognise better the extension of the thing than its intension.

Latitudines breves 2

Prosequuntur latitudines formarum:

“Quia formarum latitudines multipliciter variantur, etc.”³

Nota: titulus huius libri est latitudo formarum et subiectum ordinatur scientie speculative scilicet medie, quia subiectum eius est latitudo forme, quod est compositum ex termino naturali et mathematico. Utilitas huius liber est <ad> intelligendum quodlibet motum secundum intensum et remissum et tertium Physicorum. Etiam valet ad multa sophismata de alterationibus.

Latitudo forme est gradualis intensio forme accidentalis, sed longitudo est extensio alicuius forme secundum subiectum. Et formarum quedam est divisibilis, quedam indivisibilis. Indivisibilis est ut anima. Sed divisibilium quedam sunt divisibilis extensive ut forme brutorum animalium, alie sunt divisibiles intensive ut forme accidentales ipsius intellectus, ut videtur; quaedam sunt divisibiles extensive et intensive, ut caliditas et frigiditas; et quaedam sunt divisibiles extensive secundum extensionem subiectum, ut caliditas, alie secundum durationem, ut motus localis.

It follows the latitudes of the forms:

“Because the latitudes of the forms vary in many ways, etc”.

Note: the title of this book is “latitude of forms” and the subject is ordered under the middle speculative science, because its subject is the latitude of the form, which is composed from a natural and a mathematical term. The utility of the book is for the understanding of each motion according to the intension and remission and <for the understanding> of the third book of the *Physics*. It is also useful for many sophisms on the alterations.

The latitude of the form is the gradual intension of the accidental form, but the longitude is the extension of some form according to the subject. And among the forms, some are divisible, some indivisible. Indivisible like the soul. But among the divisible <forms>, some are extensively divisible, like the form of the animals, others are intensively divisible, like the accidental forms of the intellect, as it seems, some are extensively and intensively divisible, like warm and cold, and some are extensively divisible according to the extension of the subject, like the warm, others according to the duration, like the local motion.

Latitudines breves 3

Circa latitudines formarum.

Notandum ⟨est quod⟩ subiectum huius scientie est ly latitudo forme. Et ista scientia est media inter mathematicam et naturalem, ut patet ex subiecto. Latitudo forme est intensio forme intensibile. Formarum quedam est indivisibile ut anima intellectiva, quedam divisibile. Divisibilium quedam sunt divisibiles extensive ut forme elementorum, quedam intensive ut accidentia. Intensio forme in proposito dicitur latitudo et eius extensio ratione subiecti dicitur longitudo.

Regarding the latitude of the forms.

It is to be noted that the subject of this science is the latitude of the form. And that science is a middle science between mathematics and natural <science>, as it is evident from the subject. The latitude of the form is the intensifiable(?) intension of the form. Among the forms, some are indivisible, as the intellective soul, others are divisible. Among the divisible, some are extensively <divisible>, like the form of the elements, some intensively <divisible>, like the accidents. The “intension of form” meant here is called the latitude, and its extension according to the subject is called the longitude.

Galileo Galilei, *The Two New Sciences**

THIRD DAY, THEOREM I, PROPOSITION 1. The time in which a certain space is traversed by a moving body uniformly accelerated from rest is equal to the time in which the same space would be traversed by the same body travelling with a uniform speed (*motu aequabili*), whose degree of velocity (*velocitatis gradus*) is one-half of the maximum, final (*sumмум et ultimum*) degree of velocity of the original uniformly accelerated motion.

Let there be represented (*repraesentetur*) by extension (*extensio*) AB the time in which the space CD is traversed by a moving body accelerated from rest at point C (see Fig. 6.13). The maximum and last degree of

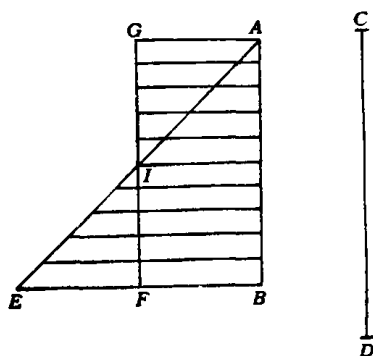


Fig. 6.13

all the degrees of velocity in the instants of time AB we let be represented by EB , constructed on AB . And, with AE drawn, all of the lines drawn from the individual points on line AB and parallel to BE will represent the increases of the degree of velocity after instant A . Then I bisect BE at F . A parallelogram $AGFB$ will be formed of the parallel lines FG , BA , and AG , BF . It will be equal to the triangle AEB , its side GF dividing AE into equal parts at I . For if the parallel lines of triangle AEB are extended up to IG , the aggregate of all the parallel lines contained in the quadrilateral will be equal to the aggregate of those contained in

* *Le Opere*, Ed. Naz., Vol. 8 (Florence, 1898), 208-12.

triangle AEB ; for those in triangle IEF are equal to those contained in triangle GIA , while those which are in trapezium $AIFB$ are common. Since each and every point on line AB corresponds to each and every instant of time AB and since the parallel lines drawn from these points and included in triangle AEB represent the growing degrees of the increasing velocity, while the parallel lines contained within the rectangle represent in the same way just as many degrees of nonincreasing but uniform velocity, it appears that there are assumed to be just as many moments of velocity (*momenta velocitatis*) in the accelerated motion represented by the growing parallel lines of triangle AEB as there are in the uniform motion represented by the parallel lines of GB . For the deficiency of velocity moments in the first half of the accelerated motion—the deficient moments being represented by the parallel lines of triangle AGI —is compensated for by the moments represented by the parallel lines of triangle IEF . It is obvious therefore that equal spaces will be traversed in the same time by two moving bodies, one of which is moved with a motion uniformly accelerated from rest, while the other is moved with a uniform motion having a moment half of the moment of the maximum velocity of the accelerated motion. Q.E.D.

THEOREM II. — If a body moves with a uniformly accelerated motion from rest, the space traversed in any given time is equal to the space traversed by a body moving uniformly with a velocity equal to the final velocity of the accelerated motion, in the same time.

Galileo Galilei *Discorsi e dimostrazioni matematiche in torno à due nuove scienze attenenti alla Meccanica et in movimenti locali*, in: *Le Opere di Galileo Galilei*, ed.. Nazionale [Dir. A. Favaro], Firenze, 1898, vol. VIII, pp. 208-209.

Engl. Transl. M. Clagett,; *The Science of Mechanics in the Middle Ages*. Madison, 1959, pp. 409-10.

